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UPGRADES TO THE PARABOLIC EQUATION MODEL

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PARABOLIC EQUATION MODEL

SAIC–88/1663

31 March 1988

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Prepared by

Eleanor Holmes
Laurie Gainey
John Hanna

Prepared for

Mr. B. Wheatley
AEAS Program Office
Code 132
ONR Deachment
NSTL Station, MS 39529

and

Dr. Martha Head
Naval Oceanographic Office

DISTRIBUTION STATEMENT A
Approved for public release;
Distribution Unlimited

Contract No. N00014–86–D–0137
Delivery Order 6

Post Office Box 1303, 1710 Goodridge Drive, McLean, Virginia 22102, (703) 821-4300

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Science Applications International Corporation

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. Agency Use Only *(Leave blank).*

2. Report Date.
31 March 1988

3. Report Type and Dates Covered.
Contract Report 31 March 1988

4. Title and Subtitle.
Upgrades To The Parabolic Equation Model

5. Funding Numbers.
Program Element No. 63785N
Project No. R0120
Task No.
Accession No.

6. Author(s).
Eleanor Holmes, Laurie Gainey and John Hanna

7. Performing Organization Name(s) and Address(es).
Science Applications International Corporation
1710 Goodridge Drive
McLean, VA 22102

8. Performing Organization Report Number.
CR N00014-86-D-0137

9. Sponsoring/Monitoring Agency Name(s) and Address(es).
ONR DET
Code 125
SSC, MS 39529-5004

10. Sponsoring/Monitoring Agency Report Number.

11. Supplementary Notes.

12a. Distribution/Availability Statement.
Approved for public release; Distribution is unlimited.

12b. Distribution Code.

13. Abstract *(Maximum 200 words).*

14. Subject Terms.
PARABOLIC EQUATION, ACOUSTIC MODELING

15. Number of Pages.

16. Price Code.

17. Security Classification of Report.
Unclassified

18. Security Classification of This Page.
Unclassified

19. Security Classification of Abstract.

20. Limitation of Abstract.

TABLE OF CONTENTS

LIST OF FIGURES

PE REVISION NOTES

INTRODUCTION

The PE model is a fully range dependent transmission loss algorithm based on a split–step Fourier solution of the standard parabolic approximation[1,2].

Under this contract, many capabilities have been added to the PE model and they are outlined in this report. Major revisions include the addition of surface loss, the addition of bottom loss, the ability to modify the pressure field at any range, the option to perform a warm start run, and several I/O options.

PE Version 2.2 was installed on the HP9050 and the SPERRY 1100/80 series computers at the Naval Oceanographic Office, Code 7300.

The main body of this document describes the changes made to PE and how they were implemented. Appendix A gives a summary and description of each of the PE modules, Appendix B contains the PE input set description, Appendix C describes the PE Pre–processor input set, and Appendix D lists sample input and output files of the sample runs described in the body of this report. Appendix E gives the format and a description of each output file that can be created by PE Version 2.2.

1.0 SURFACE LOSS

One of the major assumptions of the PE model is a flat pressure–release surface[2]. Any modifications to the surface, such as the addition of loss due to wind speed, are accounted for by applying additional attenuation to the pressure field near the surface.

In 1986, Fred Tappert suggested an approach to surface loss modelling which involved the application of additional attenuation in a thin layer near the surface[3]. This method was implemented, but the description of the algorithm contained an unknown "constant" which has not been agreed upon at this time. The implementation of the method contains an empirical constant which provides a good match to some Arctic data, but the method is not ready to be used widely without further testing. To use the Tappert surface loss method, the user inputs RMS ice roughness or wind speed as a function of range. If ice roughness is input rather than wind speed, subroutine WINDIN converts the roughness parameter to an equivalent wind speed via Equation 1–1.

$$ws = 4.94\sqrt{h\,g} \qquad \text{Equation 1–1}$$

Where :
ws = wind speed (m/sec)
h = RMS ice roughness (m)
g = 9.8 m/sec

The surface reflection loss is simulated by applying additional attenuation that is strongly localized near the surface. The attenuation decreases with depth and is applied to the pressure field until it is sufficiently negligible or until the pressure field has been attenuated at 50 mesh points in depth. In most cases the attenuation becomes negligible very quickly. This simulation is performed in the subroutine WINDMOD.

In addition to Fred Tappert's thin attenuating layer modelling of surface loss, SAIC implemented an algorithm suggested by Martha Head, which applies attenuation in wave number space at each range step. This method is described on the following pages.

1.1 PE SURFACE LOSS USING FOURIER SPACE MULTIPLICATION

IDEA:

Given a pressure field $p(z)$ in a region near the surface, we will perform a Fourier transform to obtain pressure as a function of angle $P(\theta)$, multiply the $P(\theta)$ by some surface loss table, and inverse transform back to physical space:

$$p_{sl}(z) = FFT^{-1}[\,L(\theta)\,FFT(p(z))]$$

where $L(\theta)$ is related to the loss vs angle at the surface.

ALGORITHM:

We want to define $L(\theta)$ in terms of the input surface loss table $SL(\theta)$. Since the "rays" in PE may spend several range steps in the layer near the surface, we cannot set $L(\theta)$ equal to $SL(\theta)$, causing surface loss to be applied many times for each surface reflection. Instead, we must decide how long (in number of range steps) any ray is in the vicinity of the surface, and apply the appropriate fraction of the surface loss at each step. A first approximation, and possibly the only *simple* approximation in a range-dependent environment, is a straight line ray trace. In this case, the horizontal range R_s for a ray in the surface layer is defined

$$R_s = \frac{2d_s}{\tan(\theta)}$$

where d_s is the thickness of the surface layer and θ is the incident angle of the ray at the surface. Then the number of range steps N_s a ray will be in the vicinity of the surface is

$$N_s = \frac{R_s}{dr}$$

where dr is the range step. We can now define the function $L(\theta)$ to be applied to the pressures in angle space.

$$L(\theta) = 10^{-(SL(\theta)/20./N_s)}$$

where surface loss is in dB per bounce, and $L(\theta)$ is now an amplitude.

There are two ways to perform a multiplication in angle space. The first is to perform an FFT on the pressures in physical space, multiply by $L(\theta)$ and inverse transform. The second is to convolve p(z) by the stored inverse transform of $L(\theta)$. Since the number of points in the surface layer is assumed to be small (although at this point we have not discussed it), the second method might be faster.

A quick comparison can be made to show that the first method generally will be faster. The number of multiplications involved in a convolution is N^2 where N is the number of points in the functions being convolved. The number of multiplications involved in the transform-multiply-transform is MlogM + M + MlogM where M is the power of not less than N. The following table shows the comparison for a few values of N.

N	M	2*MlogM+M	N^2
10	16	104	100
16	16	104	256
17	32	253	289

Except for very small values of N, in which computation time is small also, the faster method is the Fourier transform method.

In choosing the thickness of the surface layer, there are many factors one might consider, including frequency, angular resolution, and the sound speed profile. For this reason, the user will have complete control over the thickness of the surface layer. Until we can think of a good algorithm, there will be *no* default thickness.

IMPLEMENTATION:

In a straightforward implementation of this algorithm, we will have introduced a discontinuity into the PE pressure field. To avoid this, we will split the pressure field into

nearsurface" and "deep" using a smooth function $w(z)$ with $w(0) = 1$, $w(\infty) = 0$. Then the two parts of the pressure field are defined as

$$p_s(z) = w(z)*p(z)$$

and
$$p_d(z) = (1-w(z))*p(z)$$

where $p_s(z)$ is the near-surface field and $p_d(z)$ is the deep field. As a first cut, we will use

$$w(z) = 1 \qquad \text{for } z < d_s$$

$$w(z) = \cos^2[(z-d_s)/d_s * \pi/2] \qquad \text{for } d_s < z < 2d_s$$

$$w(z) = 0 \qquad \text{for } z > 2d_s,$$

a hanning window type transition between the bottom of the surface layer and the deep field. This forces us to re-define R_s , the horizontal range of a ray in the transition region, as

$$R_s = \frac{2d_{eff,s}}{\tan(\theta)}$$

where $d_{eff,s}$ is the effective depth of the surface layer:

$$d_{eff,s} = \int_0^{2d_s} w(z)\, dz$$

so

$$d_{eff,s} = d_s(1 + 1/2)$$

FORTRAN NOTES:

Subroutine SLOSIN will be modified to read the surface layer thickness in addition to the surface loss vs angle function.

STEP will be modified to call either subroutine SLOSS or subroutine WINDMOD on the proper flag, rather than WINDMOD only.

SLOSS , a new routine, will perform the surface loss computations.

2.0 BOTTOM LOSS

PE now models bottom loss using one of two methods. The first used when inputs are in the form of a tabulated loss vs angle ($L(\theta)$) function (often measured bottom loss data). The second method is used when the geo-acoustic sediment properties are input. A third option exists, in which the user supplies BLUG (Bottom Loss Upgrade) parameters to the PE pre-processor, which in turn creates an input set in one of the two previously mentioned formats.

2.1 GEOACOUSTIC BOTTOM INPUTS

Up to 20 bottom loss regions can be specified when describing a range-dependent geo-acoustic bottom. The geo–acoustic parameter set includes three sediment profiles that help describe bottom loss. They are: sound speed vs. depth, attenuation vs. depth, and density vs. depth. The depth-independent parameters for each bottom loss region are: shear wave velocity, the sediment-water sound speed ratio, and the thickness of the transition layer for density discontinuities.

Shear effects and density profiles are new to the geo–acoustic bottom inputs. Shear effects are controlled by the parameter CSHEAR (shear wave velocity) input on line 9C–F of the PE input set. If CSHEAR is set equal to zero then the shear effects are ignored. If CSHEAR is given a value greater than zero, however, shear effects are included by applying additional attenuation in the sediment layer.

The density profile in the sediment layer has the effect of adjusting the effective index of refraction in a thin layer near the water-sediment interface[4].

2.2 BOTTOM LOSS VS ANGLE

Loss vs. angle parameters are specified on line 9B of the PE input set. The previous restriction that loss at 0° must be 0 dB has been lifted recently with no apparent ill effects in test cases. The other restrictions in the loss vs angle curve still exist. Namely, the L(θ) curve must be monotonic, the L(θ) cannot be steeper than 1 dB per degree and the last angle in the L(θ) curve must be 90°.

2.3 BLUG PARAMETERS

The HP/PC/VAX preprocessor is now part of the PE code and adds the ability to calculate loss vs. angle and geoacoustic bottom parameters from BLUG parameters as requested in task statements #2c and #2d.

The input set for the preprocessor is quite similar to the standard PE input set up to the point where bottom loss properties are specified. At this point, the two input sets diverge, and the user should refer to the description of the inputs given in Appendix C if he wishes to access the pre–processor.

In order to invoke the code which performs the conversion of BLUG parameters to L(θ) or geo–acoustic parameters, the user requests the pre-processor option in his input set. To do this, the words "pre–processor" or "PRE–PROCESSOR" must appear in columns 61–73 of the first line (the title line) of the PE input set. If either of these words is present, PE calls its pre–processor, the BLUG inputs are read in and converted to the appropriate bottom parameters for PE.

The converted results are stored in the file PETEMP.INP (or logical unit 10 on machines such as the UNIVAC where files are not opened with explicit file names), and are fed directly to the PE code. This file is in the standard PE Version 2.2 input format and can be used as an input file to subsequent runs.

The first important difference between input sets occurs on line 9. For the standard PE input set the only variable on this line is NBOTM, but for the preprocessor the inputs are NBOTM and IBLUG. NBOTM and IBLUG are the keys to which type of inputs are to be expected. The logic is as follows :

	IBLUG = 0
NBOTM > 0 :	Read in loss v. angle for NBOTM bottom regions. (See lines 9A–9B of PE Version 2.2 input set)
NBOTM < 0 :	Read in NBOTM geoacoustic bottom regions. (See lines 9C–9F)

	IBLUG > 0
NBOTM > 0 :	Read in 11 BLUG parameters for NBOTM bottom regions and calculate loss v. angle for each region.
NBOTM < 0 :	Read in 11 BLUG parameters for each of the –NBOTM bottom regions and calculate goeacoustic bottom parameters for each region

BLUG TO L(θ)

The code and algorithms for converting BLUG parameters to loss vs angle (L(θ)) inputs to PE were borrowed from the FACT and ASTRAL models[6]. Table D–1 in Appendix D shows a typical PE input set with BLUG parameters and Table D–2 gives the resulting new PE input set.

BLUG TO GEOACOUSTIC BOTTOM:

The BLUG parameters are used first to calculate the sediment thickness which depends on the two–way travel time and the sediment sound speed profile[4]. The equation

for the sound speed in the sediment is given in Equation 2–1. The sediment sound speeds are computed at 21 equally spaced depths in the sediment layer.

$$c_i = c_0((1+\beta) \sqrt{1+2gz_i/(c_0(1+\beta))} - \beta) \qquad 2\text{–}1$$

Where:

c_0 = the sound speed at the sediment–water interface (set to 5000 ft/sec)

z_i = the depth (ft) into the sediment layer

β = the BLUG profile curvature parameter

g = the BLUG initial sound speed gradient in the sediment (1/s).

After the profile has been calculated at the 21 depths, the sediment thickness is compared to HORRAN/5. If HORRAN/5 is greater than the sediment thickness, an isovelocity layer is added to the bottom of the sound speed profile extending to the depth of HORRAN/5, the maximum sediment thickness used in the PE run.

The attenuation profile consists of two points calculated from two of the BLUG parameters (FKZ and FKZP) and the acoustic frequency. FKZ is the attenuation at the water–sediment interface (z=0) in units of (in dB/m/kHz). FKZ is multiplied by the frequency and converted to dB/ft. FKZP is the attenuation gradient (dB/m/kHz/m) and is used to calculate the attenuation at the bottom of the sediment layer. If the sediment depth is less than HORRAN/5. then an iso–attenuation layer from the sediment depth to HORRAN/5. is added.

The number of points in the tabulated density vs depth profile is always set equal to 1 to allow for a density discontinuity at the water–sediment interface, and the density at the interface is set equal to the BLUG parameter ρ_s, the sediment density. The sediment–water sound speed ratio is taken directly from the BLUG parameter RATIO.

The shear wave velocity is set to zero, since BLUG parameters provide no insight for setting this parameter.

The length in feet of the transition layer thickness for density discontinuities is also set equal to zero to allow the program to choose this length automatically.

Table D–3 shows the PE input file with BLUG parameters and Table D–4 shows the resulting converted geo–acoustic vs. bottom PE input file.

3.0 RANGE DEPENDENCE

All of the above surface and bottom treatments are range dependent, as are sound speed and bathymetry

4.0 DENSITY DISCONTINUITIES

When density discontinuities occur at the sediment–water interface the corresponding impedance mismatches have a significant effect on the transmission loss[4], due to a large change in the index of refraction. To simulate this effect without causing aliasing in PE, the density discontinuity is modeled using a hyperbolic tangent (tanh) function in a thin transition layer between water and sediment. The thickness of this layer is chosen in one of two ways. The user may allow the program to choose the thickness of the transition layer in which the smoothing is to occur, or the user may optionally select this thickness. A word of caution is in order. Choosing the transition layer thickness can be tricky for the inexperienced user, so in most cases it is best to set the transition region thickness (DENSL in the PE input file) to zero to allow the program to choose this length.

Table D–5 shows a PE input set that tests the density discontinuity modification. An iso–velocity profile is used in the water column and also in the sediment layer out to 20 nmi, with no density discontinuity in the layer. At 20 nmi the user indicates a density discontinuity and also sets the transition region thickness to 20 ft. Figure 4–1 shows the transmission loss at the 50 foot source for this run. Note that the transmission loss falls off nearly monotonically in the first 20 nmi, as we would expect with an isovelocity profile. When the density discontinuity factor is added the bottom SVP is modified (shown in Figure 4–2) and begins to turn back energy. This is reflected in the upward trend of the transmission loss beyond 20 nmi. Table D–6 gives a partial listing of the PE output file. Note the modified sediment sound speed profile at 20 nmi.

FIGURE 4–1:
TRANSMISSION LOSS, USER-SELECTED SMOOTHING LENGTH

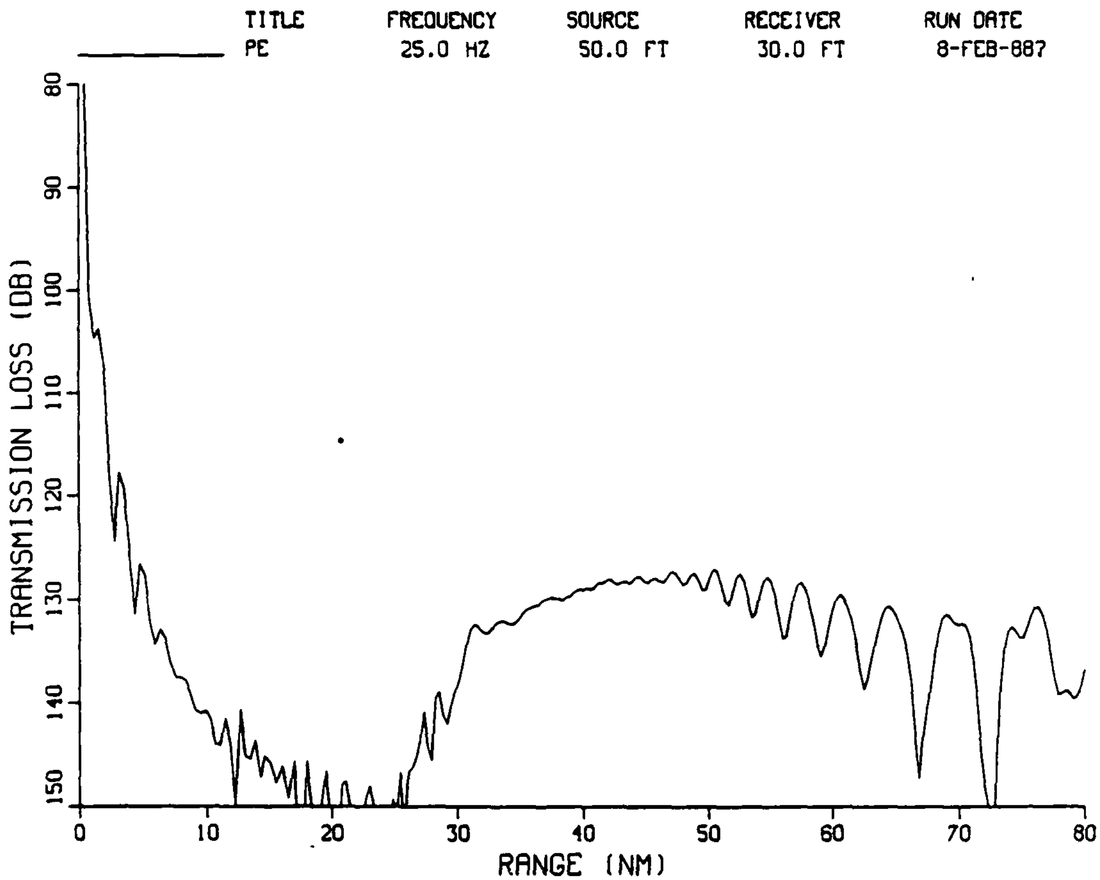


FIGURE 4–2:
BOTTOM SOUND SPEED PROFILE

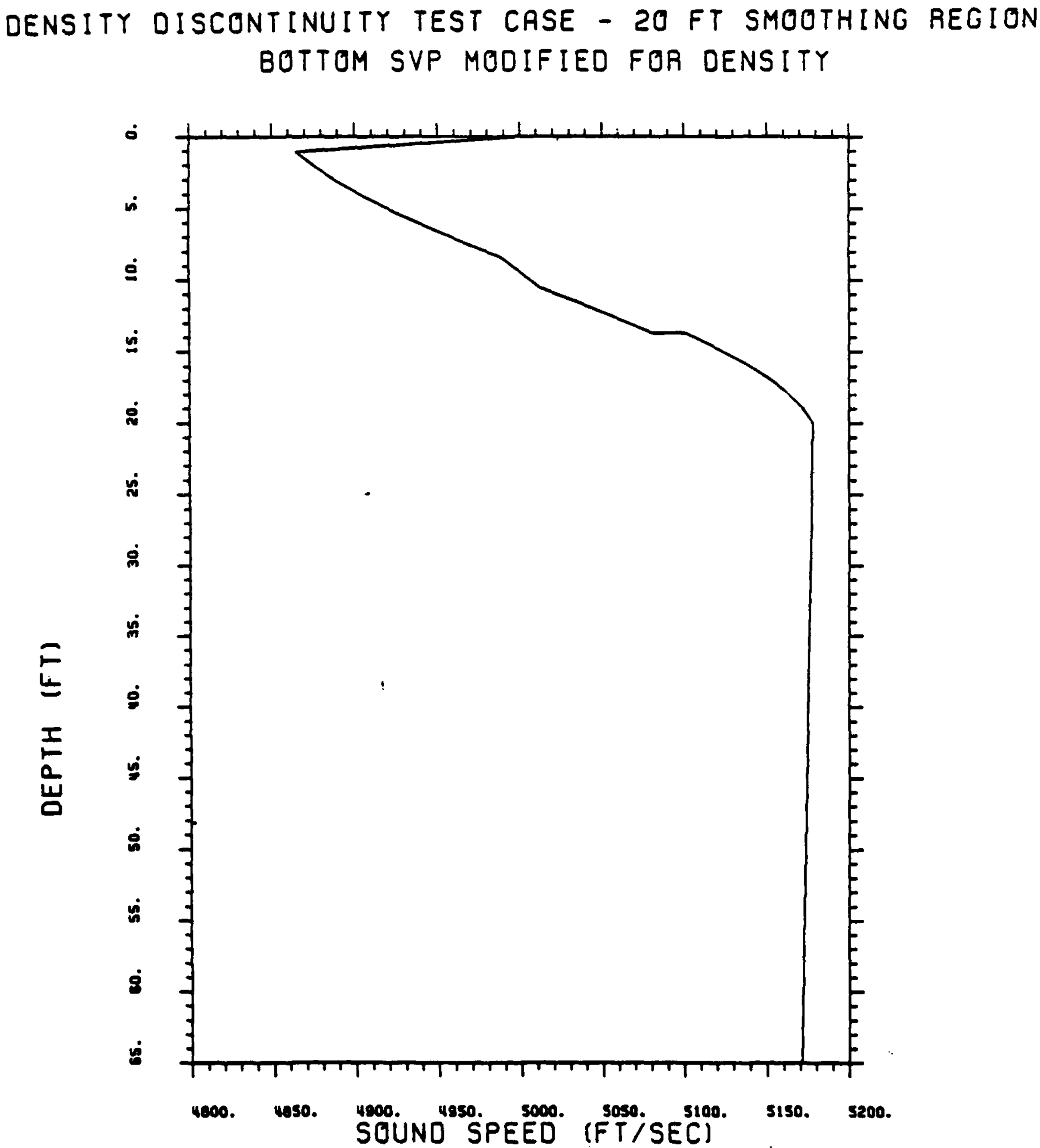


Table D–7 gives a sample input file in which the user allows the PE code to choose the transition length. The results for the 50 ft source are shown in Figure 4–3. Note that the transmission loss is less beyond 20 nmi than it was for the previous case.

The program chose a smoothing length of 64 ft. Note that the modified sediment sound speed profile (shown in Figure 4–4) is now much smoother than the SVP created with a user–selected transition length of 20 ft, and thus less energy is being turned back into the water column.

FIGURE 4–3:
TRANSMISSION LOSS FOR PE-SELECTED TRANSITION LENGTH

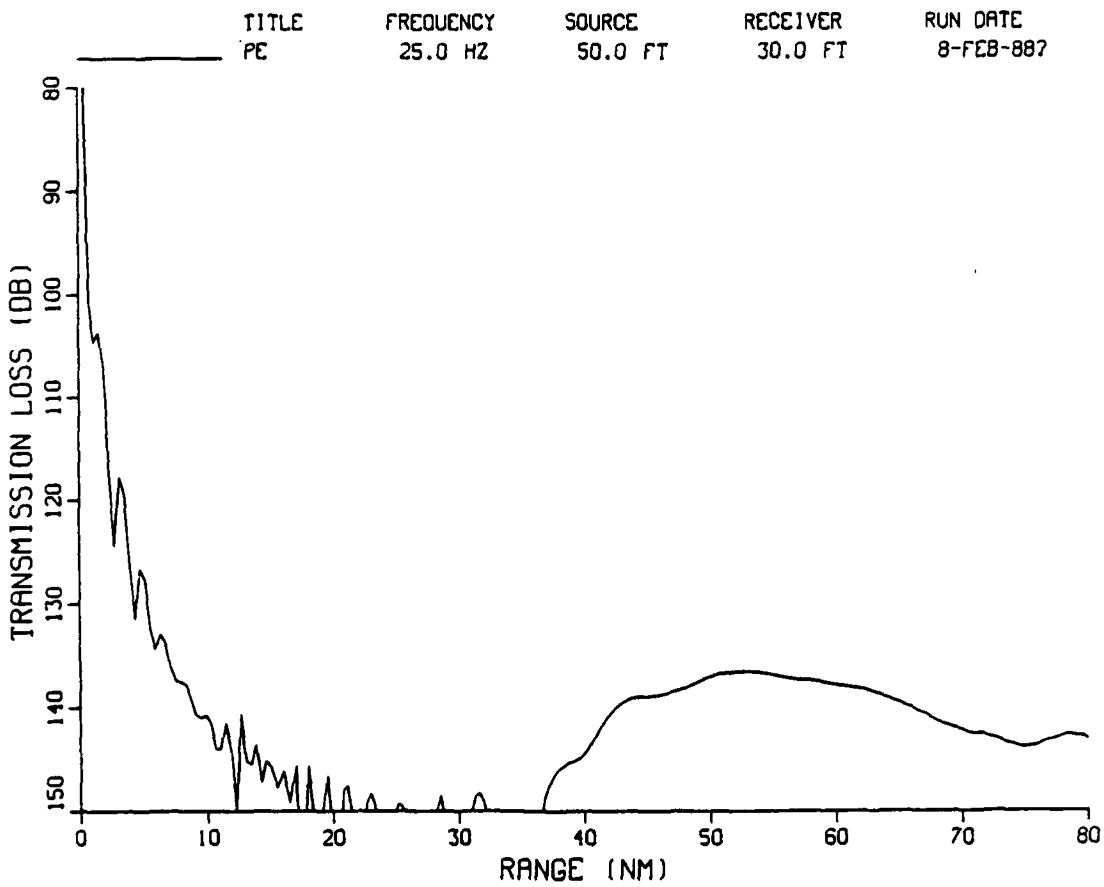
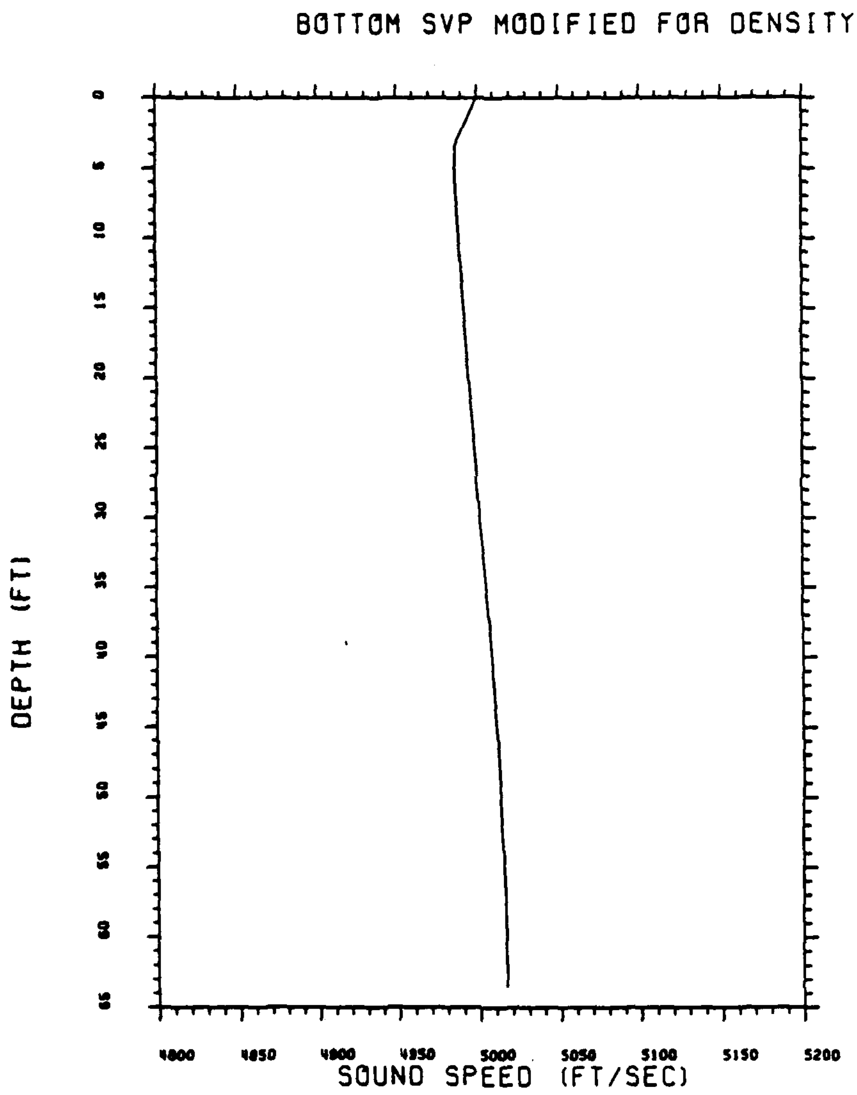


FIGURE 4–4:
BOTTOM SOUND SPEED PROFILE FOR PE-SELECTED TRANSITION LENGTH

DENSITY DISCONTINUITY TEST - PROGRAM SELECTED SMOOTHING LENGTH - 63.5 FT



5.0 FILTERING/SMOOTHING OF DEPTH DEPENDENCE

The only occurrences of filtering or smoothing in depth of the environment are the density smoothing option described in Section 4.0, and the 1–2–1 filter applied to the index of refraction table. The latter filter can be disabled using the input flag NOFILT on line 2 of the PE input set. The density smoothing can be effectively disabled by specifying a very small transition layer thickness in the between the water column and the sediment layer.

6.0 USER SPECIFIED RANGE REGIONS

This task was described as task statement #7. The user specifies a range step between user–selected ranges or can specify a block of target ranges. The default AESD–PE variable range step [1] is used at all ranges where no user–specified range step was selected.

To select a range step within a range region the user specifies a value greater than zero for NUSDR on line 2, columns 46–50 in the PE input set. NUSDR is the number of range regions where a user–defined range step is to be used. Line 2C is then repeated for each of the NUSDR range regions. On Line 2C the start and end ranges and the specified range step are input for each of the NUSDR range step regions.

Within the code the variable step size (DR) is calculated and used until RANGE+DR either falls within the bounds of a user-specified range step region or completely passes over the region. In either situation the new range step is set to (R1USDR–RANGE) where R1USDR is the range at the beginning of the next user-specified range step region, and RANGE is the current range. On following range steps the range step is set to the user–specified range step for that region until PE has advanced the solution outside or just *to* the upper bound of the range region. Due to machine precision, the upper bound of the range region may be passed by a small amount.

If the user–specified range step is greater than the variable step size a warning is issued in the PE output file. This can have an unattractive effect on the line printer field plot, but it is necessary to advise the user of possible errors associated with too large a range step. Restrictions on the range step size are virtually ignored when this option is invoked, and can lead to large errors in the calculation of the pressure field. The user should be aware of the limitations on step size and mesh spacing and the effect that ignoring these restrictions will have. Appendix F provides a summary of this topic by John S. Hanna of SAIC.

After the bounds of the range region have been passed the variable step size is used again until the next range region, if any, is encountered.

In order to choose target ranges the user inputs a value less than zero for the parameter NUSDR, and then specifies the first target range and the target range increment on line 2C–B. PE checks to see if a target range is about to be passed over and then adjusts the range step to include that target range. This option is helpful in comparing results between two runs at discrete ranges.

7.0 WARM START OPTION

This option is used to restart a run at a specified range. To use this option a number of steps must be taken and two PE runs must be made (the first run saving complex pressures for the second which is the actual warm–start run). The input variable IWRMST (line 2, column 69) controls the warm–start logic.

If IWRMST is set equal to zero then the program gracefully performs no warm–start logic. Since the warm start option uses subroutine PXTMOD, however, the user must substitute his own subroutine PXTMOD when he wishes to use the pressure field extraction/modification mode for purposes other than the warm start operations.

If IWRMST is set to 1, then the PE model will save the complex presure field to logical unit 17 at the specifed range input on line 2D–1 (read when NPXT is specified on line 2). It is advised that the target range input option be used to ensure that the pressure field is calculated and saved at precisely the desired range. Subroutine PXTMOD then opens unit 17 and writes out the pressure field at the specified range.

If IWMRST is set to 2, PE assumes that a previous run has been made with the above option, and that the pressure field is stored on unit 17. The input set for the warm start PE run must be indentical to the input set for the previous run except that IWRMST is set to 2, the variable IRS (line 2, column 68) is set to one and the starting range should be input on line 2E. The pressure field is read in via the PXTMOD subroutine and the calculations are carried out starting with the new pressure field at the input starting range.

The transmission loss values for the warm start option may differ slightly from those in the original PE run, but noticeable differences occur only at very low intensities and represent a small absolute error. This discrepency is not significant and is due to the

fact that the first range step of any PE run is only a half range step in vertical wavenumber space.

As an example, the warm start option was invoked in an environment that trapped the energy in a surface duct out to 30 nmi, where the duct disappeared, spilling energy into both CZ paths and a sub-surface duct for the next 38 Nmi. At this point, 68 Nmi from the beginning of the track, another surface duct was introduced which trapped the sub-surface ducted energy. The environment, as shown in Figure 7–1 is complex and physically unrealistic, but it serves well to illustrate the reliability of the warm start option.

FIGURE 7–1:
ENVIRONMENT FOR WARM START

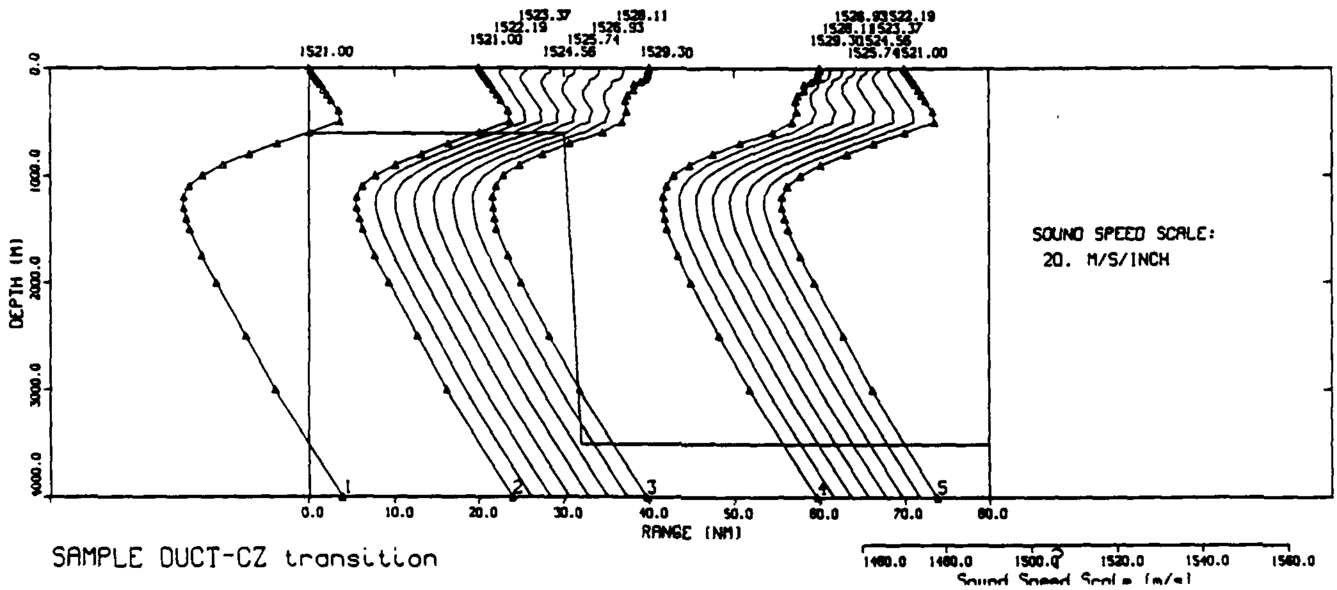
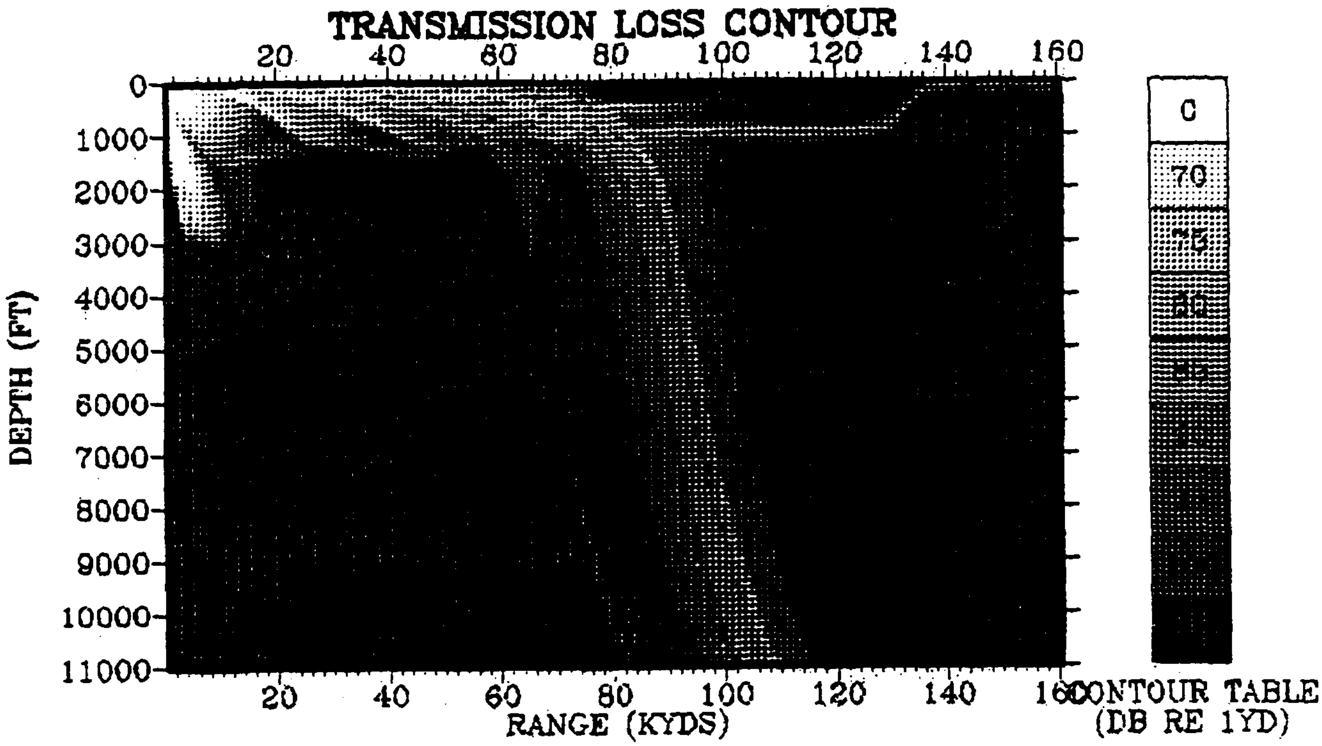


Figure 7–2 is a transmission loss contour plot, showing the loss interpolated between twenty output depths. Note the shadow zone, due to the first surface duct, below 3500 feet and out to 64 kyd (31 nmi). Beyond 64 kyds energy begins to penetrate deep into the water column where it is absorbed by the bottom, and also into a sub-surface duct. The energy in the sub-surface duct is trapped again in the surface duct at 138 kyds (68 nmi).

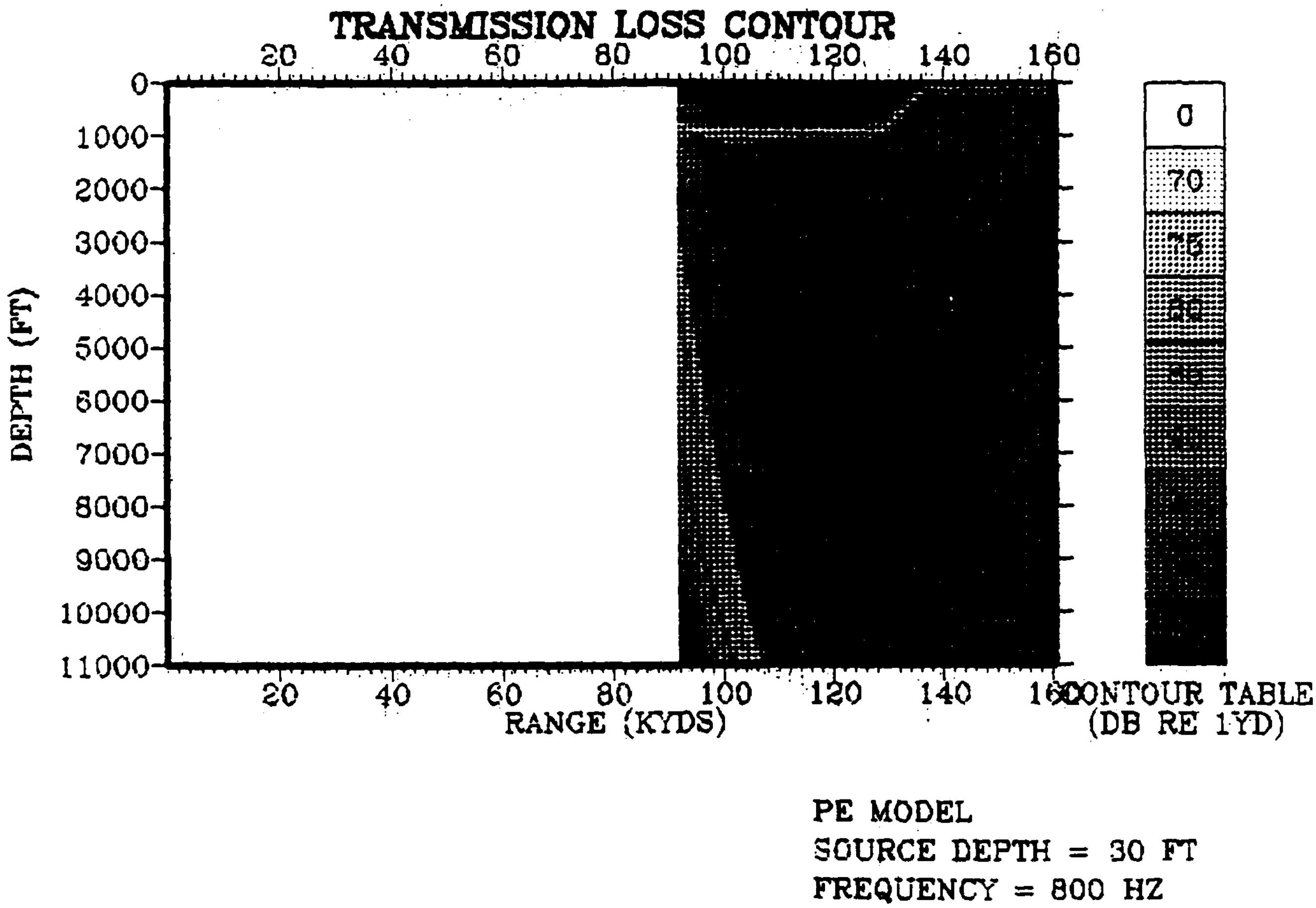
The pressure field was extracted and stored at 45 nmi (91 kyds) and then the warm start run was set up to begin at that range. Figure 7–3 shows the transmission loss contour plot for the warm start run. This PE run reproduces the results of the previous run quite accurately form 91 kyds to 160 kyds (45 to 79 nmi).

FIGURE 7–2:
TRANSMISSION LOSS CONTOUR FOR WARM START SAVE



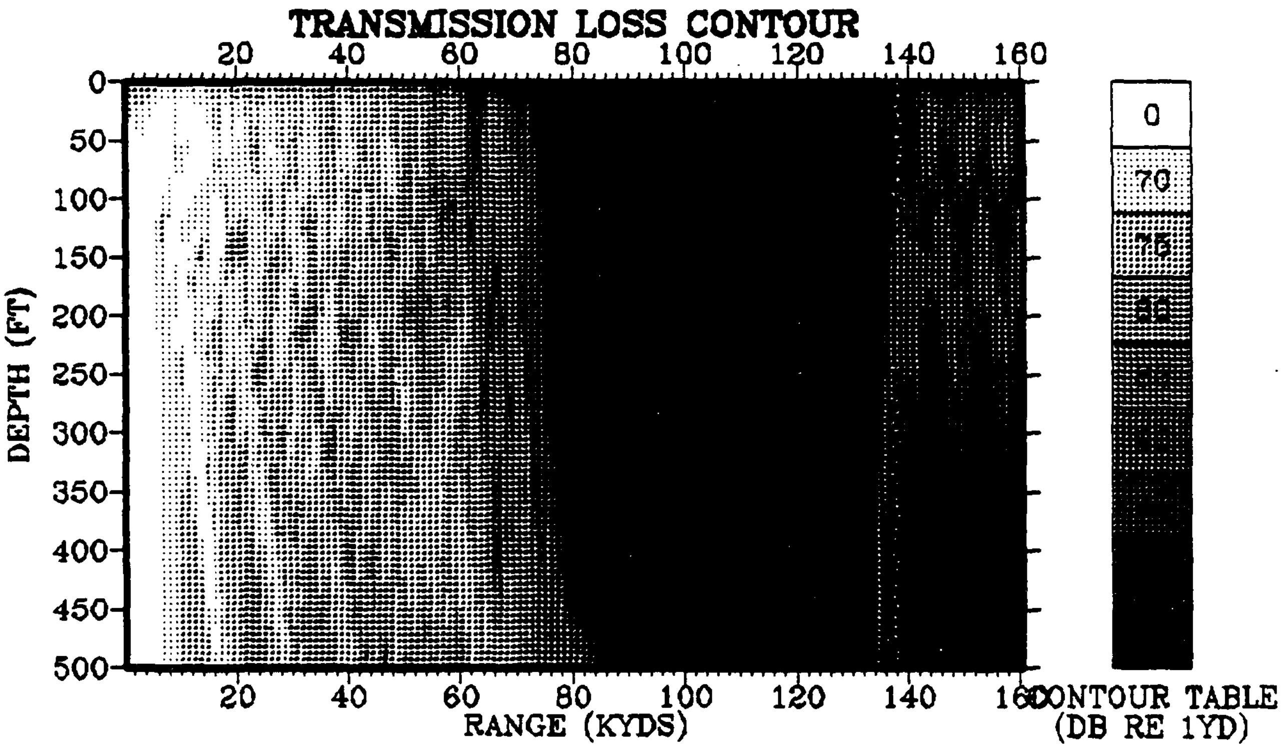
PE MODEL
SOURCE DEPTH = 30 FT
FREQUENCY = 800 HZ

FIGURE 7–3:
TRANSMISSION LOSS CONTOUR FOR WARM START RUN



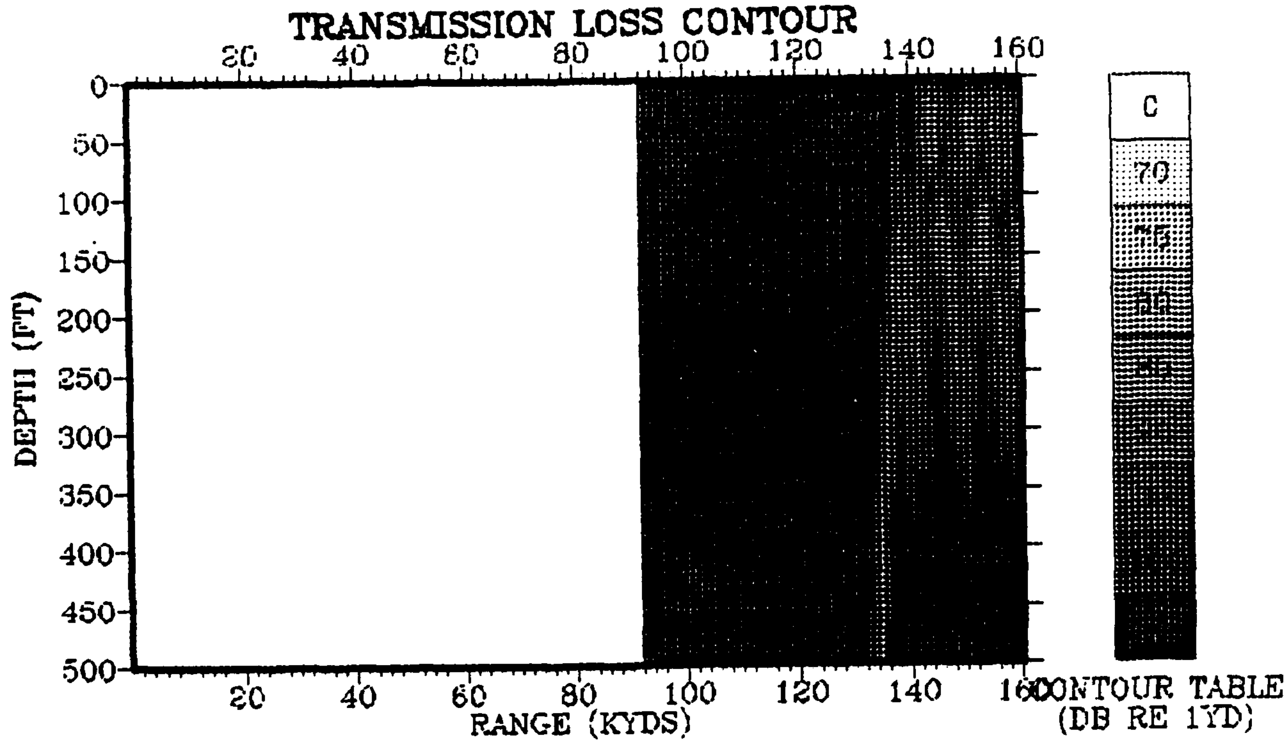
Figures 7–4 and 7–5 show close-up views of transmission loss vs range and depth in the surface duct for the original PE run and the PE warm start run, respectively. The plots show the interpolated loss between 20 output depths in the duct. Note the shadow zone in the duct between 80 and 130 kyd (40 to 64 nmi), where most of the energy is cz energy travelling deep into the water column. Again the values in Figure 7–5 closesly match those in Figure 7–4.

FIGURE 7–4:
TRANSMISSION LOSS CONTOUR FOR WARM START SAVE IN SURFACE DUCT



PE MODEL
SOURCE DEPTH = 30 FT
FREQUENCY = 800 HZ

FIGURE 7–5:
TRANSMISSION LOSS CONTOUR FOR WARM START RUN IN SURFACE DUCT



PE MODEL
SOURCE DEPTH = 30 FT
FREQUENCY = 800 HZ

8.0 PRESSURE FIELD EXTRACTION/MODIFICATION

This task is described in task statement #9 and adds the ability to extract the complex pressure field at any range or within any range region, modify it via supplementary algorithms and reinsert the modified field. To use this ability, the variable NPXT on line 2, columns 51–55 of the PE input deck must be assigned a non–zero value. If NPXT is positive then NPXT is the number of individual ranges (up to 20) at which the user can extract and modify the complex pressure field. The field will be modified at the range closest to (but not less than) the specified range. If NPXT is negative then the user will specify –NPXT range regions inside of which the pressure field will be extracted and modified at all ranges. The corresponding inputs are given on line 2D–1 if NPXT > 0 and on line 2D–2 if NPXT < 0. The extraction and modification is done via the subroutine PXTMOD. A subroutine is now supplied that performs the warm start operations and the user needs to supply his own subroutine PXTMOD in order to perform the desired modifications. The form of PXTMOD is :

```
SUBROUTINE PXTMOD(PR,PI,DZ,N,R)
```

where PR and PI are the real and imaginary parts of the pressure field, DZ is the array spacing in feet, N is the number of points in the depth mesh array and R is the current range (nmi). The pressure field is modified right before it is used to calculate the field at the next range, so the effect of modifying the pressure field at the first requested range is not reflected immediately in the output. For example, if a PE run were taking a 0.5 Nmi range step and the user doubled the values of all complex pressures at 10.0 Nmi, the transmission loss values at 10.5 Nmi and beyond, *not* at 10.0 Nmi, would reflect the 6 dB change. Also note that, since the solution for the pressure field is stepped out in range, changing the field at any range will affect the field at all subsequent ranges.

9.0 OUTPUT OPTIONS

A number of output options can be selected through the PE input set. They include the option to output transmission loss (re 1 yd) vs. range and depth, and the option to output complex pressures at all ranges for a user–specified depth window. The version of PE on the SPERRY 1100/80 additionally has the capability of saving the transmission loss (re 1 yd) at every depth mesh point and every range step.

With the exception of the standard output from PE, all output files are unformatted and all integer values are two bytes long, except on the Sperry 1100/80, where all integer values are necessarily four bytes long.

The data in these output files can be used for many purposes. For example, the transmission loss data saved in FOR041.DAT (logical unit 41) was used to create the transmission loss vs. range plots given in this report (e.g. Figure 4–1), and the transmission loss contour plots (e.g. Figure 7–2). The pressure field data saved in FOR044.DAT (logical unit 44) could be used to perform beamforming at a selected depth at many ranges.

Each of the output files contains at least one header record which provides some of the necessary run parameters, followed by data records which contain the data relevant to the individual output file. For example, transmission loss will be saved to unit 41 at each output depth and every range if the variable ND on line 2 columns 1–5 is given a non–zero value. The pressure field is interpolated to the depths specified on line 5, converted to transmission loss in dB re 1 yard, and written to logical unit 41.

A description of each output file and its format is provided in Appendix E.

10.0 REFERENCES

1 H.K. Brock, "The AESD Parabolic Equation Model", AESD TN–75–07, ONR, Arlington, VA.,(1978)

2 F.D. Tappert, "The Parabolic Approximation Method", in Wave Propagation and Underwater Acoustics, edited by J.B. Keller and J.S. Papadakis, (Springer–Verlag, New York, 1977), Lecture Notes in Physics.

3 F.D. Tappert, "Parabolic Equation Modeling of Surface Loss", SAIC Preprint, (May, 1986)

4 C.W. Spofford, W.W. Renner and H.J. Venne, "Modification to FACT and ASTRAL for the Bottom–Loss Upgrade", SAI–84–148–WA, (April, 1983)

APPENDIX A
PE VERSION 2.1
DRIVER and SUBROUTINE DESCRIPTIONS

ORGANIZATION OF THIS APPENDIX

The PE model driver and subroutines are organized into five categories. They are called input, important computational, support computational, side computational and output routines.

The input routines read inputs, mainly from the user, but in one case from another file. The important computational subroutines carry the PE solution out in range. Every important computational routine has the potential to modify the values of PR or PI, the real and imaginary parts of the complex pressure field. Support computational routines provide results for use by the important computational routines, but do not advance the solution themselves. Side computational routines perform computations which do not affect the PE solution. Function TLOSS, for example, computes transmission loss from the complex pressures, but does not affect (or help to advance) the PE solution. Output routines, naturally, write PE results out to files. Some routines would seem to fall into more than one category, and these have been classified according to their major function.

The categories will be presented in the order mentioned above, and within each category, the subroutines will be arranged in alphabetical order. There are two exceptions to this rule. First, the driver, PE, will be the first in its category, input routines. Second, the routines in the important subroutines category will be arranged approximately in the order in which they are used in PE.

INPUT ROUTINES

PE (MAIN routine)

The main routine, PE, reads and prints out the user inputs (excluding bottom loss information), opens appropriate files, calls PETL to compute transmission loss, and prints the transmission loss vs range.

PE calls various subroutines to read inputs such as sound speed profiles and bathymetry. The bottom loss inputs are read by PETL. All of the input subroutines described here are called directly by PE.

GETBOT

GETBOT reads in the user–specified bottom depth vs range.

RDPROF

RDPROF reads in the user–specified sound speed profiles when provided in the PE input set, and creates FORTRAN unit 2, an unformatted file containing the sound speed profiles for internal use. Since local variables are in English units, RDPROF converts metric sound speed profiles to English units. If the user has not input sound speed profiles, he must provide FORTRAN unit 2 to PE.

If the spherical earth correction is to be applied, RDPROF makes the correction:

$$Z <= Z(1+Z/4181\times10^4)$$
$$C <= C\left(\frac{2090\times10^4}{2090\times10^4-Z}\right)$$

RTHIN

If the user has specified a shaped source, RTHIN reads in level vs angle for the source function.

SLOSIN

SLOSIN reads in loss vs angle in each surface loss region.

USDRIN

USDRIN reads in either the range regions where the user wishes a user-specified range step, or the target ranges for PE.

WINDIN

WINDIN reads and prints out wind speed or ice roughness as a function of range. For each region that the user has specified an ice roughness rather than wind speed, WINDIN converts the roughness parameter into an equivalent wind speed

$$V_{ice,equivalent} = 4.94\sqrt{hg}$$

where h is the rms ice roughness in meters and g is 9.8 m/s^2.

IMPORTANT ROUTINES

PETL

Strictly speaking, PETL should not be included in the important subroutines category because the pressure field is not modified in PETL. However, PETL is the main subroutine in PE, and controls the range loop and the flow of most of the other important subroutines, so it is appropriate to include it here.

Phase error and attenuation: Prior to the range loop, PETL must set up certain constants for the PE run. First, the flags for the high angle phase velocity correction are defined. The choices for this correction are CMOD, Thomson–Chapman, or no correction at all.

Next, the volume attenuation is defined. If the user has not specified an attenuation profile as a function of depth, the volume attenuation is considered to be constant and equal to:

$$\alpha = 0.125f^2 \text{ if } f<1 \text{ kHz}$$

$$\text{and } \alpha = 2f^2\left(\frac{.1}{1+f^2} + \frac{40}{4100+f^2}\right) \text{ for } f>1 \text{ kHz}$$

where f is frequency in kHz and α is volume attenuation in dB/Nmi. If the user has specified an attenuation vs depth table, PETL calls subroutine PHATTEN to interpolate the user-supplied $\alpha(z)$ table onto the PE depth mesh.

Computing transform sizes: The PE depth mesh is a function of the frequency, bottom depth, the source half-beamwidth, and maximum angle of interest. There are two transform sizes computed for the PE run. The first is called the source transform size, and the second is called the PE transform size. The PE directed source is computed using the source transform size, and the PE solution is advanced using the PE transform size. In the case of a non-directed source, these two transform sizes are equal.

First, the necessary mesh spacing for the source beamwidth is computed. Given this mesh spacing, the extent of the isovelocity absorbing layer is increased so that the total PE depth mesh is a power of 2 times the mesh spacing, thus defining the source transform size. This provides for a source of the exact beamwidth requested. If the user has requested a directed source, the maximum angle of interest will be larger than the source half-beamwidth, and a second transform size is computed, providing the "effective" beamwidth

for the problem. Example: A particular directed source from 3 to 10 degrees has been requested, so the source half-beamwidth will be 3.5 degrees, requiring a 2^7 source transform size (for this frequency and bottom depth). However, the effective beamwidth must be at least 10 degrees in order to propagate the energy in the directed source, and the PE transform size will be 2^9 in this case.

All of the parameters associated with the mesh spacing are computed at this point.

Sediment: PETL reads the inputs associated with the sediment properties for this PE run. The user may specify either a geo-acoustic bottom or an L(θ) table. In the case of the L(θ) bottom, PETL calls subroutine LOSGEN to generate the appropriate sound speed profile and attenuation table in the sediment layer. If a geo–acoustic bottom is specified, subroutine DNSITY or DNSTY2 is called to modify the bottom sound speed profile to take into account the sediment density, and subroutine SHEAR is called to produce the additional attenuation needed in the sediment to account for loss due to shear wave conversion. All input and computed parameters for each range region in the bottom are stored on a scratch file to be read as the program steps out in range.

Range loop setup: PETL first reads through the sediment files to find the correct bottom parameters for the user-specified starting range. Next, subroutine FILTER is called to set up the index of refraction at the starting range. Subroutine SOURCE is then called to create the complex pressures at the starting range. SOURCE also creates the array FIL, the filter roloff function, and subroutine FILLOS is called by PETL to add sediment attenuation to this array.

Range loop: At each range step, PETL computes the bottom index, calls STEP to advance the solution, calls the output routines TLOSS and FLD, and performs bookkeeping on the range-dependent properties of the environment before proceeding to the next range step. The bookkeeping consists of checking for new sound speed profiles and new bottom loss regions. At each new sound speed profile, subroutines FILTER and INDEX are called to create the index of refraction and the modified index of refraction table respectively. At each new bottom loss region, subroutine FILLOS is called to modify the array FIL, the filter roloff function used to attenuate energy both at high angles and in the isovelocity absorbing layer.

SOURCE

Subroutine SOURCE is called by PETL to create the initial pressure field at range zero. The mathematics of the SAIC PE digital source function are discussed fully in the PE/ASTRAL CODE DOCUMENTATION[1].

The modification not discussed in the PE/ASTRAL document is the option for a shaped source function. If the user has specified a level vs angle source function, SOURCE calls RTH to multiply the level vs angle by the source function amplitude in wave number space. This feature has been included to aid in modeling receivers (PE sources) on the bottom in situations where the source level at any angle is the coherent sum of two "rays", one which has travelled through the bottom to the receiver and one which has reached the receiver before hitting the bottom.

RTH

Subroutine RTH, called by SOURCE, multiplies complex pressures in wave number space by a user–supplied level vs angle function.

STEP

STEP is called by PETL to advance the PE solution one range step:

$$\psi(r+\Delta r,z) = e^{i\Delta r k_0(n^2-1)/2}\,\mathrm{FFT}^{-1}\left[\,e^{-i\Delta r K^2/2k_0}\,\mathrm{FFT}(\psi(r,z))\right]$$

where Δr is the range step, z is depth, k_0 is the reference wave number, n is the index of refraction, and K is the vertical wave number.

FFT(ψ(r,z)) : On entry to STEP, ψ is defined in physical space (z–space) on the PE depth mesh. The exception to this is the first range step, where ψ already has been transformed by subroutine SOURCE into vertical wave number space (k–space). Therefore, on all range steps but the first, STEP performs a sine transform on PR and PI, the real and imaginary parts of ψ on the PE depth mesh. The result is $\psi(K,z)$, the complex pressure field in k–space.

PE range step: After transforming the complex pressure field into wave number space, STEP computes the range step accommodating the various user range step directives. The first of these is the fixed range step, in which case STEP does no computation, but assigns the user input fixed range step size to Δr, the PE range step. In

the case of a variable range step, STEP computes

$$\Delta r = \lambda/(1-\cos(\theta))$$

where λ is the acoustic wavelength and θ is the angle steeper than which there is no significant energy propagating at this range. If the user has specified target ranges, Δr is defined as the minimum of the computed Δr and the distance to the next target range.

K–space step : STEP now multiplies the complex pressures by the second derivative transform table, performing the bookkeeping necessary for the case of a new range step. If the range step has not changed from the previous step, the complex pressures are multiplied by the stored second derivative transform table created by subroutine SET.

When there is a new range step, STEP computes and multiplies the pressure field by

$$\mathrm{FIL}^{*}e^{-i(\Delta r_1/2+\Delta r_2/2)K^2/2k_0}$$

where Δr_1 is the old range step and Δr_2 is the new range step. STEP then calls subroutine SET to create $\mathrm{FIL}^{*}e^{-i\Delta r_2 K^2/2k_0}$ on the PE depth mesh for subsequent range steps (until Δr changes again). In both of these cases, FIL is applied only at angles larger than the effective PE beamwidth. If a sediment layer is present, STEP also calls FILLOS, which modifies the filter roloff array FIL on the mesh points corresponding to the sediment, where $\mathrm{FIL} = e^{-\alpha(z)\Delta r}$, α being the attenuation at depth z .

$\mathbf{FFT^{-1}(\psi(K,z))}$: Before performing the inverse FFT, STEP checks to see if the user wishes a Fourier interpolation for transmission loss at the PE output depths. If this is the case, $\psi(K,r)$ is saved in temporary arrays by subroutine PSAVE for use by subroutine TLOSS. After the pressures have been saved, the inverse sine transform is performed, again by RST.

z–space step : The complex pressures are now multiplied by the index of refraction operator and volume attenuation,

$$\mathrm{FIL}^{*}e^{i\Delta r(n^2-1)/2}\, e^{-\alpha\Delta r}$$

or

$$\mathrm{FIL}^{*}e^{i\Delta r(n-1)}\, e^{-\alpha\Delta r} ,$$

the second operator being used when the Thomson–Chapman high angle correction is being used. STEP handles the case of range-dependent bottom depth by sliding up or down the parts of the stored index of refraction arrays associated with the sediment layer and the

absorbing layer. FIL is applied in the sediment and in the isovelocity absorbing layer.

Surface loss : Fred Tappert's surface loss algorithm is implemented at this point by applying an additional attenuation to pressures near the surface. Subroutine WNDMOD performs this task.

PXTMOD

PXTMOD is a user–supplied subroutine called at user–specified ranges or range intervals. The complex pressures as a function of depth are available to PXTMOD to be examined and/or modified.

A version of PXTMOD is available which will perform a warm start for PE. That is, the user may stop a PE run at a certain range, save the complex pressure field, and re–start the PE run at this range at a later date.

RST

Subroutine RST performs a sine or cosine transform. Subroutine SOURCE uses the RST to perform a cosine transform on the initial impulse response function H(z) . STEP calls RST to transform the complex field back and forth between physical space and wave number space using the sine transform.

WNDMOD

WNDMOD applies an additional attenuation to the pressures near the surface to simulate loss due to surface roughness.

SUPPORT ROUTINES

DNSITY

Subroutine DNSITY is used to compute the modified sediment sound speed profile given a density discontinuity at the water–sediment interface and constant density in the sediment layer.

DNSTY2

DNSTY2 computes the modified sediment sound speed profile given a piece-wise linear density profile in the sediment layer, assuming no discontinuities, even at the water-sediment interface.

ELI2

ELI2 computes the elliptical integral of the second kind using a series summation with a Landen transform. ELI2 is called by INTEG2.

FILLOS

FILLOS multiplies the sediment layer attenuation, $e^{-\alpha(z)\Delta r}$, by the filter roloff array FIL. α is the attenuation in the sediment at depth z, z ranging from the depth at the water–sediment interface to the depth at the bottom of the sediment layer. Δr is the range step.

FILLOS is called by PETL at each new bottom loss region, and is called by STEP each time the range step size changes.

FILTER

FILTER computes the index of refraction, n^2-1 on the PE depth mesh and saves the results for subroutine INDEX. If the CMOD high angle correction is being used, FILTER modifies all depths in the water column:

$$z <= z\ (\sqrt{c_0/c(z)}\ )$$

before computing n^2-1.

FNMUD

Function FNMUD computes n^2-1 in the sediment layer between the water column and the absorbing layer.

INDEX

INDEX computes U, used by STEP to advance the PE solution in physical space.

$$U = FIL\ e^{-\alpha\Delta r} e^{i\Delta r k_0 (n^2-1)/2}$$

is computed on the PE depth mesh, where FIL is the filter roloff function created by SOURCE to attenuate the energy in the isovelocity absorbing layer, $e^{-\alpha\Delta r}$ is the volume attenuation in the water column, and $e^{i\Delta r k_0 (n^2-1)/2}$ is the index of refraction table. Δr is the range step, k_0 is the reference wave number and n is the index of refraction. These quantities are computed for the water column and the sediment and absorbing layers.

In the case that the Thomson–Chapman high angle phase correction is used, the (n^2-1) must be replaced by $2(n-1)$, and since (n^2-1) is passed as a single variable FN into INDEX from subroutine FILTER, $2(n-1)$ is obtained by inverting FN in subroutine INDEX.

The results from INDEX are stored in the complex array (UR,UI) and used by STEP.

INTERP

INTERP is called by subroutine RTH to interpolate the function $R(\theta)$ from the user-defined angle grid onto the PE k–space mesh.

INT3

INT3 is called by INTERP to linearly interpolate a function f(x) to a point x_0.

INTEG1 and INTEG2

Since the attenuation profile in the sediment is assumed to be a piecewise-linear function,

$$\alpha_i(z) = c_i + d_i z$$

equation 2.2.7 in "Incorporation of an Ocean Bottom in the Parabolic Equation Model"[2] can be divided into segments corresponding to the linear segments of α, so

$$L_i(\hat{z}) = c_i I_i(\hat{z}) + d_i J_i(\hat{z})$$

where

$$I_i(\hat{z}) = 2 \int_0^{\hat{z}} [(a^2 + \hat{z}^2 - z^2)/(\hat{z}^2 - z^2)]^{\frac{1}{2}}\, dz$$

and

$$J_i(\hat{z}) = 2 \int_0^{\hat{z}} z\, [(a^2 + \hat{z}^2 - z^2)/(\hat{z}^2 - z^2)]^{\frac{1}{2}}\, dz$$

with $a = \pi/\mathrm{HORRAN}$, HORRAN being the horizontal distance traveled by all rays in the bottom. $J_i(\hat{z})$ is solved analytically by function INTEG1. $I_i(\hat{z})$ can be expressed as the difference of two elliptic integrals of the second kind, and INTEG2 calls subroutine ELI2 to evaluate the elliptic integrals. Both INTEG1 and INTEG2 are called by LOSGEN.

LOFZ

LCFZ is called by LOSGEN to perform a table look-up and return bottom loss in dB as a function of turning depth in the sediment.

LOSGEN

LOSGEN generates the piece-wise linear function $\alpha(z)$ where z is depth into the sediment and α is loss in dB/foot. LOSGEN first checks the input $L(\theta)$ profile to make sure that there are no gradients steeper than 1 dB/degree. Then a depth mesh of break points, or layers, is defined at intervals of 250 feet in the sediment *and* at every depth which is the turning point of a ray whose grazing angle is included in the input $L(\theta)$ table.

After this initial bookkeeping, LOSGEN loops though this depth mesh computing the $\alpha(\hat{z}_i)$ at each depth. Inside the depth loop, LOFZ is called to convert turning point

depth $\hat{z}_i$ to grazing angle and to then perform a table look-up and return bottom loss for the ray which has turning depth $\hat{z}_i$. Functions INTEG1 and INTEG2 provide $J(\hat{z}_i)$ and $I(\hat{z}_i)$ respectively. From these, LOSGEN solves for $\alpha(z)$ in each layer, using the results from the previous layers and the fact that α is linear and continuous. If in any layer the computed α is negative, the layer is bisected, inserting an extra break point, and α is re-computed. If the bisection fails five times in a row to produce a non-negative value for α, an error message is issued and the program is aborted.

PHATTEN

Subroutine PHATTEN is called by PETL to interpolate a user-supplied attenuation vs depth table onto the PE depth mesh.

SEDTHK

SEDTHK computes sediment thickness given an initial gradient and sound speed, the sound speed curvature parameter β, and two way travel time.

SET

SET computes the second derivative transform table passed to subroutine STEP as the complex variable S = (SR,SI),

$$S = FIL * e^{\frac{i\Delta r}{2k_0}k^2} .$$

where Δr is the range step, k_0 is the reference acoustic wave number and k is the vertical wavenumber. FIL is the filter roloff function which smoothly eliminates angles larger than the PE effective beamwidth in k–space, and eliminates energy in the isovelocity absorbing layer in z–space.

SHEAR

Shear, called by PETL, modifies the attenuation vs depth function in the bottom to account for loss due to shear wave conversion.[2]

SPEED

Function SPEED, called by various routines, returns the sound speed at the requested depth at the current range.

SVP

Subroutine SVP reads two records from FORTRAN unit 2. The first contains the sound speed profile for the current range. The second contains the starting range for the next sound speed profile.

SIDE ROUTINES

TLOSS

Function TLOSS computes transmission loss at one user–specified output depth. TLOSS is called by PETL at each range and each output depth. Transmission loss in dB re 1 yard is

$$TL = 10 \log_{10}(I**2*(9/R))$$

where I is the intensity at the output depth and R is the range in feet. The "9" references intensity to 1 yard, and is necessary because internal distances are in feet. Since P is assumed to include only vertical spreading and not cylindrical spreading, the range term includes this effect.

The intensity is computed in one of two manners. By default, the intensity is defined to be the weighted average of the intensities at the two bracketting PE mesh points. If the user has requested a Fourier interpolation for intensity, however, subroutine INVERSE computes the complex pressure (PR_f, PI_f) using Fourier interpolation. The intensity I is then $PR_f^2 + PI_f^2$.

Additional intensity may be included in the transmission loss computation if the user has set the "steep angle" flag. In this case, subroutine STEEP is called to perform a short-range, steep angle ray trace for rays steeper than the PE beamwidth.

INVERSE

Subroutine inverse, called by TLOSS, performs the inverse discrete Fourier transform to interpolate for complex pressure between mesh points.

PSAVE

PSAVE is called by subroutine STEP whenever the user has requested a Fourier interpolation for transmission loss at any output depth. PSAVE copies the complex pressure arrays in wave number space into temporary arrays in common for use by subroutine INVERSE.

OUTPUT ROUTINES

FLD

FLD, called by PETL, draws the line printer field plot. On the first call to FLD the depth axis and header are drawn. Then, either every range step or every 1 Nmi, depending on input set specifications, FLD draws one line of field plot data, with different symbols corresponding to different transmission loss levels.

PRPROF

If the user has supplied sound speed profiles on FORTRAN unit 2, but wishes the profiles to be printed out anyway, PRPROF is called to read and print out these profiles.

REFERENCES FOR APPENDIX A

1 E.S. Holmes, "PE/ASTRAL Code Documentation" SAI–83–735–WA

2 R. Steiglitz, L. Dozier, H.M. Garon, and C.W. Spofford, "Incorporation of an Ocean Bottom into the Parabolic Equation (PE) Algorithm", SAI–79–878–WA, (January, 1979)

APPENDIX B
PE INPUTS

The following pages contain a description of the standard PE version 2.2 input set. This appendix is taken from the draft of the Software Product Specification for the Parabolic equation model.

PE Version 2.2 STANDARD INPUT SET

As an aide to creating a PE input set, the following general description of each line and its inputs is provided:

LINE	VARIABLE(S)
1	Title
2	Flags and special inputs (i.e., non–zero starting ranges)
3–5	Geometry of the problem (source–rcv positions, max range, etc)
6	Sound speed profiles
7	Bathymetry
8–9	Sediment description

UPGRADE HISTORY SINCE VERSION 2.0

Version 2.0 to Version 2.1 The only modifications made to PE were the addition of attenuation due to shear wave conversion in the bottom and user control over the transition region length for density discontinuities. The inputs associated with both are input on line 9C. Line 2 and 2F input formats have been rearranged slightly.

Version 2.1 to 2.2 If the user includes the word "PRE–PROCESSOR in columns 61–73 of his PE title (line 1), PE will immediately call subroutine PREP to read an alternate input set and create a PE version 2.2 input set. This change allows users to create PE input sets with a comfortable format while making no changes to the basic PE code. Subroutine PREP currently reads BLUG parameters and system correction factors in addition to the normal version 2.1 inputs. Associated with the BLUG parameters is the "tenth" parameter, the power of frequency upon which attenuation is dependent.
A new input has been added, with which the user can specify an output range step. That is, the user can specify that he wants transmission loss output to the transmission loss file and the printed output every 5 Nmi, instead of whatever range step PE chooses. This input is entered on line 3.
Inputs and logic have been added to the PE code so that the ICECAP surface loss models can be accessed. See lines 2 and 2A.

LINE 1: (A60,A13)

COLUMNS	VARIABLE	DESCRIPTION
1–60	TITLE	Title for this run
61–73	PRFLG	If PRFLG is set to "PRE–PROCESSOR" or to "pre–processor", then subroutine PREP is called immediately to create a PE version 2.1 formatted input set from the rest of *this* input set. If you set PRFLG to call subroutine PREP, do not proceed with this document, but refer to the input file description for the pre–processor, "PE VERSION 2.2 PRE–PROCESSOR INPUTS" for the rest of this input set. *DEFAULT*: " "

LINE 2: (13I5,5I1)

COLUMNS	VARIABLE	DESCRIPTION
1–5	ND	Number of output depths. ND must not be greater than 20. On a PC with a line printer line width of about 80, 11 is the maximum number of output depths whose printed TL output will fit neatly across a page. The PE output depth is the *moving* point, and the input depth is the *stationary* point. *DEFAULT* : *no default* *RANGE* : [*1 to 20*] *RECOMMENDATION*: *as many as are needed. The number of output depths does not affect the run time.*
6–10	IFLAT	Number of bathymetry points. PE can accept up to 501 (range,depth) pairs for bathymetry. *DEFAULT* : *0* *RANGE* : [*0 to 501*] *RECOMMENDATION*: *as many points as there are available. More points provide more accuracy with no cost in run time.*
11–15	IPRNT	Print flag for transmission loss. Set IPRNT to 1 for a printout of TL vs Range at selected output depths. *DEFAULT* : *0* *RANGE* : [*0 and 1*]
16–20	NPLT	Print flag for line printer field plots. Set NPLT to the number of columns you want in the field plot. Set NPLT to 0 for *no* field plot. Set NPLT less than 0 to save a file (unit 3) containing TL vs Range and Depth for later use. For example, NPLT=–120 will plot (and save) 120 columns of TL vs Range and Depth. (See line 2, column 52 and line 4, columns 31–70 for more field plot information.) *DEFAULT* : *0* *RANGE* : [–12*0 to 120*] *RECOMMENDATION*: *70 unless the user wants output file size kept to a minimum, in which case 0 is recommended*

21–25 ISPH

Spherical earth correction flag. Set ISPH to 1 for no spherical earth correction.

DEFAULT : *0*
RANGE : *[0 and 1]*
RECOMMENDATION : *0*

26–30 NTRANS

Transform size selector. Use NTRANS to *artificially* increase the transform size. Set NTRANS to the power of 2 (i.e. 11, not 2048) desired. Note that the transform size cannot be reduced but only increased.

DEFAULT : *0*
RANGE : *[0 to 12]*
RECOMMENDATION : *0*

31–35 NTHB

Number of points in Shaped Source function. The shaped source option is used to simulate a near-bottom receiver (PE stationary point) or to specify a receiver beam pattern. The shaped source is input on line 2A.

DEFAULT : *no shaped source*
RANGE : *[0 to 91]*
RECOMMENDATION : *an adequate sampling of the shaped source function in angle space. If the user does not wish to examine each beam pattern, 91 points from −45 to 45 degrees are recommended*

36–40 NCDR

If NCDR = 0, PE checks and possibly adjusts the range step size at each range step. If NCDR is non-zero, the range step is checked and adjusted only every NCDR range steps. This option saves a bit of execution time.

DEFAULT : *every range step checked*
RANGE : *[0 to 9999]*
RECOMMENDATION : *0*

41–45 NWIND

NWIND is the number of surface loss regions in range. If NWIND > 0, wind speed or ice roughness regions are specified. In this case the wind speeds and ice roughness parameters are input on line 2B–1 and possibly 2B–1a. If NWIND < 0, loss vs angle is input on line 2B–2 .

DEFAULT : *0*
RANGE : *[−20 to +20]*
RECOMMENDATION : *[−20 to 0]. Again, increas-*

		ing the number of surface loss regions does not increase the run time.
46–50	NUSDR	Set the NUSDR flag to the number of range regions where a user–defined range step is to be used. Up to 20 range regions are allowed. Line 2C–1 contains the range regions and associated range steps. Set NUSDR to a negative number to input –NUSDR target ranges rather than range step regions. In this case, target ranges are entered on line 2C–2 . *DEFAULT* : *0* *RANGE* : [*–9999 to +20*] *RECOMMENDATION*: *Target ranges (–9999 to 0)*
51–55	NPXT	If NPXT is positive, then NPXT is the number of *individual* ranges, up to 20, at which the user can extract, examine and/or modify the complex pressure field and then return to the program. Further inputs for this option are on line 2D–1. If NPXT is negative, then –NPXT is the number of range *regions* inside of which the pressure field can be modified or extracted at *every* range step. Inputs for NPXT < 0 are on line 2D–2. *DEFAULT* : *0* *RANGE* : [*–20 to +20*] *RECOMMENDATION*: *0*
56–60	NRBEAM	Number of ranges at which complex pressures at user–specified depths (see line 2F–1) are to be saved for beamforming. If NRBEAM > 0, then individual ranges are entered on line 2F–1. If NRBEAM < 0, a minimum range and delta–range are entered on line 2F–2. *DEFAULT* : *0* *RANGE* : [*–9999 to 20*] *RECOMMENDATION*: *application-dependent*
61–65	NPH	Number of points in a user-supplied attenuation vs depth function. Set NPH > 0 to input attenuations in dB/Nmi and depths in feet. Set NPH < 0 to input attenuations in dB/km and depths in meters. The attenuation vs depth profile is entered on line 2G. *DEFAULT* : *0* *RANGE* : [*0, 2 to 100*] *RECOMMENDATION*: *0*

66	NOFILT	Set NOFILT to 1 to disable the 1–2–1 filter used to smooth the index of refraction table. *DEFAULT* : *0* *RANGE* : [*0 and 1*] *RECOMMENDATION*: *0*
67	IDENSE	Set IDENSE to 1 for a field plot every range step instead of every 1 Nmi. If IDENSE is set and NPLT < 0, the TL vs Range and Depth also will be saved every range step instead of every Nmi. *DEFAULT* : *0* *RANGE* : [*0 and 1*] *RECOMMENDATION*: *0*
68	IRS	IRS > 0 indicates a starting range other than 0 . The starting range will be input on line 2E if IRS has been set. *DEFAULT* : *0* *RANGE* : [*0 and 1*] *RECOMMENDATION*: *application-dependent*
69	IWRMST	Set IWRMST to zero for a normal PE run. A warm start PE run is a run in which the user starts the PE run at a range greater than zero, using the complex pressures saved from a previous PE run as an initial condition. If you wish to save the complex pressures for a future warm start PE run, set IWRMST to 1, set NPXT (columns 51–55 of this line) to 1, and enter the range at which pressures are to be saved on line 2D–2. For a warm start PE run, assuming you have already run PE with IWRMST set to 1 and have saved the complex pressures, set IWRMST to 2, set NPXT to 1, and enter warm start range on line 2D–2 as described above. Additionally, set IRS (column 68) to 1 and enter the starting range on line 2E . *DEFAULT* : *0* *RANGE* : [*0 to 2*] *RECOMMENDATION*: *0*
70	ISTEEP	Since PE propagates energy in a limited aperture, the transmission loss especially at short ranges may be artificially high (too much loss). If ISTEEP is set to 1, subroutine STEEP is called whenever transmission loss is output, and intensities are supplemented with the help of a straight line ray trace to points outside the PE beamwidth.

DEFAULT	: *0*
RANGE	: [*0 and 1*]
RECOMMENDATION	: *1*

***** INCLUDE LINE 2A O'LY IF NTHB > 0 *****

LINE 2A	(8F10.2)	
COLUMNS	VARIABLE	DESCRIPTION
1–80	(TTHB(I),RTHB(I),I=1,NTHB)	

The level vs angle function for a near–bottom source (PE stationary point). There are four (angle,level) pairs per line. In using this option, the implementer should have a thorough enough knowledge of signal processing to understand the effect of shaping a source with a simple multiply in wave number space. Shaping a source arbitrarily can cause aliasing and/or relocation in depth of the source.

DEFAULT : *none. If you need to include this line, you must specify a level vs angle function.*

RANGE : *angle [−90. to 90.]*
level [0. to +9999.]

RECOMMENDATION : *keep this curve as smooth as possible*

***** ENTER LINES 2B–1 ONLY IF NWIND > 0 *****
***** REPEAT LINE 2B–1 FOR EACH OF THE
NWIND SURFACE LOSS REGIONS *****

<u>LINE 2B–1</u> (F10.2,I5,5X,4F10.2,I5)

<u>COLUMNS</u>	<u>VARIABLE</u>	<u>DESCRIPTION</u>
1–10	RWIND(I)	Range in Nmi of the start of the Ith surface loss region *DEFAULT* : *none* *RANGE* : [*0. to 25000.*]
11–15	NOSL(I)	Flag indicating an ice or wind region. If NOSL(I) is 0, an air-water interface is assumed at the surface and the wind speed in columns 21–30 is used. If NOSL(I) is between 1 and 8, an ice-water interface is assumed, and ice roughness parameters in columns 31–65 are used. The ice loss models are: NOSL(I) <u>MODEL</u> 1 Low Frequency Burke-Twersky 2 Low Frequency Free Surface 3 Marsh-Mellen 4 Mid Frequency Rigid Surface 5 Asymptotic Burke-Twersky 6 Gordon-Bucker 7 Buck-Wilson 8 Default *DEFAULT* : *air-water interface* *RANGE* : [*0 to 8*] *RECOMMENDATION* : *0 or 8*
21–30	WIND(I)	Wind speed, entered in knots if WIND(I) > 0, and in meters/second if WIND(I() < 0. This variable may be left blank or zero if NOSL(I) ≠ 0. *DEFAULT* : *0.* *RANGE* : [*–999. to 999.*]
31–40	SIGMA(I)	RMS Ice roughness in feet for this surface loss region, used only if NOSL(I) ≠ 0. If SIGMA(I) < 0, ice roughness is assumed to be in meters. *DEFAULT* : *0.0* *RANGE* : [*0. to 999.*] *RECOMMENDATION* : *0.0*

41–50	SEP(I)	Keel spacing or ridge separation in feet, used only if NOSL(I) $\neq$ 0. If SEP(I) < 0, keel spacing is assumed to be in meters. *DEFAULT* : *328.* *RANGE* : [*0. to 99999.*] *RECOMMENDATION*: *328., if unknown*
51–60	GBFACT(I)	Fudge factor for Gordon-Bucker model, used only if NOSL(I) $\neq$ 0 *DEFAULT* : *0.75* *RANGE* : *0. to 1.* *RECOMMENDATION*: *0.75*
61–65	ITYPE(I)	Used only in the Gordon-Bucker model, ITYPE defines the additional attenuation due to seasonal variation. If ITYPE is 0, no additional attenuation is applied. If ITYPE is 1, Summer attenuation is included, and if ITYPE is 2, Winter attenuation is included. *DEFAULT* : *0* *RANGE* : *0 and 1* *RECOMMENDATION*: *Season-dependent*

***** ENTER LINES 2B–2 ONLY IF NWIND < 0 *****
***** REPEAT LINES 2B–2a AND 2B–2b
FOR EACH OF THE –NWIND SURFACE REGIONS *****

LINE 2B–2a (2F10.2,I5)

COLUMNS	VARIABLE	DESCRIPTION
1–10	RSLOSS(I)	Starting range in Nmi for the Ith surface loss region. DEFAULT : *0., but 0 range can be specified for only the first range region* *RANGE* *[0. to 25000.]* *RECOMMENDATION* *see recommendation for NWIND input*
11–20	DLAYER(I)	Surface layer thickness in feet for the Ith surface loss region. PE will use 8 mesh points as the minimum layer thickness. *DEFAULT* : *8 mesh points* *RANGE* : *[0. to max water depth]* *RECOMMENDATION* : *0.*
21–25	NSLOSS(I)	Number of points in the Ith surface loss region. *DEFAULT* : *none* *RANGE* : *[2 to 30]* *RECOMMENDATION* : *as many points as are needed to describe the surface loss curve*

LINE 2B–2b (8F10.2)

COLUMNS	VARIABLE	DESCRIPTION
1–80	(RSL(J,I),SL(J,I),J=1,NSLOSS(I))	Enter four (angle, surface loss) pairs per line. Angles are in degrees, loss is in dB per bounce. *DEFAULT* : *none* *RANGE* : *angle [0. to 90.] loss [0. to 300.]*

***** ENTER LINE 2C–1 ONLY IF NUSDR > 0 *****
***** REPEAT LINE 2C–1 FOR EACH OF THE NUSDR RANGE REGIONS *****

LINE 2C–1 (3F10.2)

COLUMNS	VARIABLE	DESCRIPTION
1–10	R1USDR(I)	Starting range in Nmi for the Ith user–specified range step region. *DEFAULT* : *0.* *RANGE* : [*0. to 25000.*]
11–20	R2USDR(I)	End range in Nmi for the Ith user–specified range step region. *DEFAULT* : *none, R2USDR(I) must be greater than R1USDR(I)* *RANGE* [*>0. to 25000.*]
21–30	DRUSDR(I)	User–specified range step in Nmi for the Ith range step region. Note: In between user specified range step regions, PE will use the default variable range step or DR, the user–supplied range step from line 4. *DEFAULT* : *none* *RANGE* : [*>0. to 25000.*] *RECOMMENDATION*: *do not use this option without a good idea of what the maximum "safe" range step is (see SPS for PE for range step definition). The program will, however, issue a warning if the user-specified range step is larger than the computed range step.*

***** ENTER LINE 2C–2 ONLY IF NUSDR < 0 *****

LINE 2C–2 (2F10.2)

COLUMNS	VARIABLE	DESCRIPTION
1–10	R1USDR	Initial target range in Nmi. *DEFAULT* : *0.* *RANGE* : [*0. to 25000.*] *RECOMMENDATION*: *none*

11–20	DRUSDR	Target range increment in Nmi. PE will adjust the variable range step so that the –NUSDR ranges, R1USDR, R1USDR+DRUSDR, R1USDR+2*DRUSDR, etc, are "hit". That is, these ranges will occur at the *end* of a PE range step, not somewhere in the middle. *DEFAULT* : *0.* *RANGE* : [*0. to 25000.*] *RECOMMENDATION*: *something greater than 0.*

***** ENTER LINE 2D–1 ONLY IF NPXT > 0 *****

<u>LINE 2D–1</u>	(8F10.2)	
<u>COLUMNS</u>	<u>VARIABLE</u>	<u>DESCRIPTION</u>
1–80	(RPXT(I),I=1,NPXT)	

Ranges in Nmi at which the pressure field is extracted, modified by the user-supplied subroutine PXTMOD, and returned. Subroutine PXTMOD will be called at the range closest to (but not less than) RPXT(I). The form of PXTMOD is:
SUBROUTINE PXTMOD(PR,PI,DZ,N,R)
where PR and PI are the real and imaginary parts of the pressure field, DZ is the depth spacing of the array in feet, N is the number of points in the arrays, and R is the current range in Nmi. PXTMOD is called at the beginning of each range step from physical space (depth space).

The PXTMOD supplied by SAIC with PE version 2.2 and later will perform the warm start (see line 2, column 69) complex pressure storage and recovery.

DEFAULT : *none*
RANGE : *[0. to 25000.]*
RECOMMENDATION: *do not use this option*

***** ENTER LINE 2D–2 ONLY IF NPXT < 0 *****

<u>LINE 2D–2</u>	(8F10.2)	
<u>COLUMNS</u>	<u>VARIABLE</u>	<u>DESCRIPTION</u>
1–80	(RPXT(I),RPXT2(I),I=1,NPXT)	

Pairs of ranges in Nmi bracketting regions where complex pressure field is to be extracted and/or modified. At every range step between each pair (RPXT(I),RPXT2(I)), the pressure field is send as described on line 2D–2 to subroutine PXTMOD.

DEFAULT : *none*
RANGE : *RPXT(I) [0. to 25000.]*
RPXT2(I) must be greater than RPXT(I)
RECOMMENDATION: *do not use this option*

***** ENTER LINE 2E ONLY IF IRS > 0 *****

LINE 2E	(F10.2)	
COLUMNS	VARIABLE	DESCRIPTION
1–10	RSTART	Starting range for this PE run. This option is useful when an environment has complex sound speed profiles and/or bathymetry, and saves the user from having to set up the "track" for each starting range. RSTART also is used for a warm start (see line 2, column 69) PE run. If the user has performed a PE run in which he has saved the complex pressures at 100 Nmi for a future warm start, he probably will want to start *this* PE run at 100 Nmi and save the execution time needed to reach 100 Nmi. *DEFAULT* : *0.* *RANGE* : [*0. to 25000.*]

***** ENTER LINE 2F ONLY IF NRBEAM ≠ 0 *****

<u>LINE 2F–1</u> (2F10.2)

COLUMNS	VARIABLE	DESCRIPTION
1–10	Z44MIN	Minimum depth in feet for saving complex pressure field. *DEFAULT* : *0.* *RANGE* : *[0. to (max bottom depth +sediment thickness)*4/3]* *RECOMMENDATION*: *0., if disk space is unlimited. Otherwise, see SRS Paragraph 3.2.1 for a definition of the size of the complex pressure file created*
11–20	Z44MAX	Maximum depth in feet for saving complex pressure field. *DEFAULT* : *0.* *RANGE* : *[0. to (max bottom depth +sediment thickness)*4/3]* *RECOMMENDATION*: *see Z44MIN*

***** ENTER LINE 2F–2a ONLY IF NRBEAM > 0 *****

<u>LINE 2F–2a</u> (8F10.2)

COLUMNS	VARIABLE	DESCRIPTION
1–80	(R44(I),I=1,NRBEAM)	Individual ranges in Nmi at which the pressure field is saved for beamforming. *DEFAULT* : *0.* *RANGE* : *[0. to 25000.]* *RECOMMENDATION*: *list all interesting ranges*

***** ENTER LINE 2F–2b ONLY IF NRBEAM < 0 *****

<u>LINE 2F–2b</u>	(2F10.2)	
<u>COLUMNS</u>	<u>VARIABLE</u>	<u>DESCRIPTION</u>
1–10	R44MIN	Minimum range in Nmi at which the complex pressures are saved for beamforming. *DEFAULT* : *0.* *RANGE* : [*0. to 25000.*] *DEFAULT* : *first range of interest*
11–20	DR44	Range increment for saving complex pressures. The complex pressures will be saved at the –NRBEAM ranges closest to but not less than R44MIN, R44MIN+DR44, R44MIN+2*DR44, etc . *DEFAULT* : *0.* *RANGE* : [*0. to 25000.*]

***** ENTER LINE 2G ONLY IF NPH ≠ 0 *****

LINE 2G	(4(F10.2,F10.5))	
COLUMNS	VARIABLE	DESCRIPTION
1–80	(ZPH(I),APH(I),I=1,ABS(NPH))	Four (depth, attenuation) pairs per line for the user-specified attenuation vs depth profile. If NPH > 0 , units are feet and dB/Nmi . If NPH < 0 , units are meters and dB/km . The user may supply up to 50 points in this profile. *DEFAULT* : *depth none* *attenuation 0.* *RANGE* : *depth [0. to 35000.]* *attenuation [–5000. to +5000.]* *RECOMMENDATION*: *Do not use this option*

LINE 3 (8F10.2)

COLUMNS	VARIABLE	DESCRIPTION
1–10	ZS	PE input depth (source is stationary point) in feet. *DEFAULT* : *none* *RANGE* : [*>0. to 100000.*]
11–20	F	Frequency in Hz. *DEFAULT* : *none* *RANGE* : [*1. to 100000.*], *although if the computed transform size is too large, PE will issue a warning and halt. The selection criteria should preselect those frequencies appropriate for PE.*
21–30	BEAM	Half–beamwidth for PE run. This is the steepest angle of interest. If no directed source is specified, this is also the half-beamwidth of the PE source, and PE will extend the isovelocity absorbing bottom layer so that the source half–beamwidth is *exactly* BEAM. To prevent the isovelocity absorbing layer from being extended, the user can set NTRANS on line 2 (to a number less than the transform size PE would choose anyway). Any time NTRANS is set, PE guesses that the user may want control over the PE mesh spacing, and therefore does nothing that would change that mesh spacing. *DEFAULT* : *20.* *RANGE* : [*0. to 45.*] *RECOMMENDATION* : *20.*
31–40	C0	C0 is the reference sound speed in ft/sec *or* m/sec. There are three options associated with C0. The first is obtained by setting C0 to 0.0 . This enables CMOD, the sound speed/depth modification to propagate high angles with greater accuracy. The second, the Thompson–Chapman high angle correction, is chosen by setting C0 to –1.0 . Finally, if C0 > 0.0, neither high angle correction is made, and energy with a phase velocity other than C0 will be propagated incorrectly. *DEFAULT* : *CMOD* *RANGE* : [*–1. to +100000.*]

		RECOMMENDATION: *–1.*
41–50	VABSF	VABSF is set to 1.0 to *disable* volume attenuation. To include volume attenuation, set VABSF to 0.0 . *DEFAULT* : *0.* *RANGE* : [*0. and 1.*] *RECOMMENDATION*: *0.*
51–60	THMIN	If a directed source is to be used, set THMIN to the minimum angle for the directed source. That is, if the desired source is from –10 to +20 degrees, then set THMIN=–10.0 . *DEFAULT* : *–BEAM* *RANGE* : [*–45. to +44.*] *RECOMMENDATION*: *D/E angle minus half-beamwidth*
61–70	THMAX	Maximum angle for directed source. If the user wants a source symmetric around 0 degrees, then both THMIN and THMAX should be set to 0.0 . *DEFAULT* : *+BEAM, if THMIN is set to zero. No default if THMIN is non-zero* *RANGE* : [*> THMIN to 45.*] *RECOMMENDATION*: *D/E angle plus half-beamwidth*
71–80	DROUT	User-specified range step in Nmi for output of transmission loss to the unformatted transmission loss file and to the printed output (if printed transmission loss was requested on line 2). This input does not affect the internal PE range step, but controls the size of output files. The PE transmission loss will be output at the next range step after each multiple of DROUT. For example, if DROUT is 5 Nmi, then the next range step after 5, 10, 15, Nmi, etc will be output. To get transmission loss at *exact* multiples of DROUT, use the target range inputs described on line 2. *DEFAULT* : *Normal PE range step* *RANGE* : [*0. to 25000.*] *RECOMMENDATION*: *Output range step of interest. This input has NO effect on PE accuracy.*

LINE 4 (8F10.2)

COLUMNS	VARIABLE	DESCRIPTION
1–10	DMAX	Maximum depth in feet for the entire PE run. If bathymetry is provided and DMAX is less than the deepest bathymetry point, DMAX will be adjusted by the program. However, if DMAX is greater than the deepest bathymetry point, PE will assume that the user wants to control the mesh spacing, and will use DMAX instead of the deepest bathymetry point in computing the PE transform size. *DEFAULT* : *none, unless bathymetry is supplied, in which case PE chooses the largest depth in the input bathymetry table* *RANGE* : [>*0. to 100000.*]
11–20	RMAX	Maximum range in Nmi. *DEFAULT* : *none* *RANGE* : [*0. to 25000.*]
21–30	DR	User–selected fixed range step for the entire PE run. To have PE use a variable range step, set DR to 0.0 . This is *strongly* recommended. If a fixed range step is taken, and is chosen to be too large, PE will produce inaccurate results **without** warning the user. *DEFAULT* : *0.* *RANGE* : [*0. to 25000.*] *RECOMMENDATION*: *0.* !!!!
31–40	CD1	CD1 is the minimum depth in feet for the line printer field plot and for the file containing TL vs range and depth (see line 2, columns 16–20, variable NPLT). If NPLT is set to 0, CD1 is not used by PE. *DEFAULT* : *0.* *RANGE* : [*0. to 100000.*] *RECOMMENDATION*: *0.*
41–50	CD2	CD2 is the maximum depth in feet for the line printer field plot and for the file containing TL vs range and depth. If NPLT is set to 0, CD2 is not used. *DEFAULT* : *none* *RANGE* : [>*CD1 to 100000.*] *RECOMMENDATION*: *maximum depth of interest*

51–60	CLMIN	CLMIN is the minimum transmission loss in dB for the line printer field plot. If NPLT is set to 0, CLMIN is not used. *DEFAULT* : *0.* *RANGE* : [*0. to 300.*] *RECOMMENDATION*: *80.*
61–70	DCL	DCL is the transmission loss increment for the line printer field plot. There are 5 transmission loss levels displayed in the field plot. For example, if CLMIN=70 and DCL=10, then the 5 levels with symbols will be: < 70 dB 70–80 dB 80–90 dB 90–100 dB and > 100 dB *DEFAULT* : *0. dB, not an interesting plot* *RANGE* : [*0. to 300.*] *RECOMMENDATION*: *10.*
71–80	BCORR	Occasionally the user wishes to "see into" the sediment layer or the bottom in the PE line printer field plot. BCORR is the number of feet into the bottom that the field plot will print transmission loss symbols instead of "B's". *DEFAULT* : *0.* *RANGE* : [*0. to 100000.*] *RECOMMENDATION*: *0.*

LINE 5	(8F10.2)	
COLUMNS	VARIABLE	DESCRIPTION
1–80	(D(I),I=1,ND)	Up to 20 output depths in feet. *DEFAULT* : *none* *RANGE* : [>*0. to 100000.*] *RECOMMENDATION*: *all depths of interest, as increased number of output depths does not result in an increase in run time.*

LINE 6 (2I5)

COLUMNS	VARIABLE	DESCRIPTION
1–5	NPROF	Number of sound speed profiles input. If profiles are to be input from the alternate sound speed file, for002.dat, then set NPROF to 0 . If the alternate method is used to input profiles and you want a printout of these profiles, set NPROF to –1 . *DEFAULT* : *program expects profiles already on unformatted file, unit 2.* *RANGE* : [*–1 to +9999*] *RECOMMENDATION*: *number of sound speed profiles available on track*
6–10	NOPRO	Flag used to inhibit the printing of the input sound speed profiles in the PE standard output file. Set NOPRO to 1 to prevent the sound speed profile printout. *DEFAULT* : *print profiles* *RANGE* : *0 and 1*

***** ENTER LINES 6A AND 6B ONLY IF NPROF > 0 *****
***** REPEAT LINES 6A AND 6B FOR EACH OF THE NPROF PROFILES *****

LINE 6A (F10.2,I5)

COLUMNS	VARIABLE	DESCRIPTION
1–10	RANGE(I)	Range in Nmi of the Ith sound speed profile. *DEFAULT* : *0.* *RANGE* : [*0. to 25000.*]
11–15	NPTS(I)	Number of points in the Ith sound speed profile. *DEFAULT* : *none* *RANGE* : [*2 to 50*] *RECOMMENDATION*: *as many points, less than 50, as are available*

LINE 6B (8F10.2)

COLUMNS	VARIABLE	DESCRIPTION
1–80	(Z(II),C(II),II=1,NPTS(I))	Depth, sound speed pairs, four per line, for the Ith sound speed profile. Depths and sound speeds may

be in ft and ft/sec or in meters and meters/sec.

DEFAULT	: *none*
RANGE	: *depths* [*0. to 100000.*] *sound speeds* [*1000. to 10000.*]

***** ENTER LINE 7 ONLY IF IFLAT > < 0 *****

LINE 7	(8F10.2)	
COLUMNS	VARIABLE	DESCRIPTION
1–80	(BR(I),BZ(I),I=1,ABS(IFLAT))	Bathymetry, in four (range,depth) pairs per line. If IFLAT > 0, range is entered in Nmi and depth is entered in feet. If IFLAT < 0, range is in Nmi and depths are in meters. *DEFAULT* : *none* *RANGE* : *ranges [0. to 25000.]* *depths [>0. to 100000.]* *RECOMMENDATION*: *all available bathymetry points*

LINE 8 (F10.2,I5,5X,2F10.2)

COLUMNS	VARIABLE	DESCRIPTION
1–10	HORRAN	HORRAN indicates the kind of bottom used for this PE run. First, if HORRAN is set to 0.0, then the bottom is assumed to be fully absorbing and **NO FURTHER INPUTS ARE NEEDED AFTER LINE 8.** Secondly, if HORRAN is not 0.0, then PE provides a sediment layer between the water and the absorbing basement. In this sediment layer, the user may describe a geo–acoustic bottom (in which the user supplies attenuation and sound speed as a function of depth in the sediment) or provide a tabulated loss vs angle function. The thickness of this geo-acoustic sediment layer is HORRAN/5 . For a loss vs angle type bottom, the recommended value for HORRAN is 6000 ft in deep water and 2000 ft in shallow water, and HORRAN represents the horizontal cycle distance in the bottom of energy, independent of grazing angle. The sediment layer will turn back rays up to 30 degrees grazing angle. For a geo–acoustic bottom, HORRAN should be set to 5 times the sediment thickness in feet, with regard to the fact that PE running times increase linearly with an increase in the total water+sediment depth. *DEFAULT* : *0.* *RANGE* : [*0. and 2000. to 6000.*] *RECOMMENDATION*: *0. for absorbing bottom, 6000. for deep ocean, 2000. for shallow water*
11–15	IPRCNT	Part of the PE depth mesh computation takes into account the sound speed profile in the sediment and the depth extent over which the gradient is approximately constant. IPRCNT defines the term "approximately". IPRCNT is the percentage change allowed in sound speed gradient in the sediment over the depth region of "constant" gradient. This input is not used normally in implementation, since it applies only to a geo-acoustic bottom. *DEFAULT* *not used* *RANGE* : [*0 to 1000*] *RECOMMENDATION*: *0*

21–30	DBDOWN	DBDOWN is used in the PE range step computation. The PE range step is defined as $$\frac{\lambda}{1-\cos(\theta)}$$ where λ is the reference wave length, and θ is the angle above which all energy is more than DBDOWN dB below the peak energy level at the current range. *DEFAULT* : *10 dB* *RANGE* : [*0. to 50.*] *RECOMMENDATION*: *0.*
31–40	DRMAX	Maximum range step in feet. This input can be useful in very low frequency cases, where PE normally would take large range steps, and the user wishes transmission loss values at a dense set of ranges. *DEFAULT* : *3038.05 ft* (*1/2 Nmi*) *RANGE* : [*>0. to 60761.* (*10 Nmi*)] *RECOMMENDATION*: *0.* (*defaults to 1/2 Nmi*)]

LINE 9 (I5)

COLUMNS	VARIABLE	DESCRIPTION
1–5	NBOTM	NBOTM is the number of bottom regions for either a geo–acoustic or a loss vs angle description of the bottom. To choose a loss vs angle description, make NBOTM > 0. To choose a geo–acoustic bottom, set NBOTM < 0. That is, for 5 geo-acoustic bottom regions, NBOTM is –5 . *DEFAULT* : *none* *RANGE* : [*–9999 to –1, 1 to 9999*] *RECOMMENDATION*: *as many bottom regions as there are available*

***** ENTER LINES 9A AND 9B ONLY IF NBOTM > 0 *****
***** REPEAT LINES 9A AND 9B FOR EACH OF THE NBOTM REGIONS *****

<u>LINE 9A</u> (F10.2,I5)

<u>COLUMNS</u>	<u>VARIABLE</u>	<u>DESCRIPTION</u>
1–10	RANGE(I)	Range in Nmi of the beginning of the Ith bottom region. *DEFAULT* : *0.* *RANGE* : *[0. to 25000.]*
11–15	NTHETA(I)	Number of points in the tabulated loss vs angle curve for this bottom region. This function may contain up to 50 points. *DEFAULT* : *none* *RANGE* : *[2 to 50]* *RECOMMENDATION* : *as many points as there are available*

<u>LINE 9B</u> (8F10.2)

<u>COLUMNS</u>	<u>VARIABLE</u>	<u>DESCRIPTION</u>
1–80	(THETA(II),LOSS(II),II=1,NTHETA(I))	Tabulated loss vs angle curve for this bottom loss region, four pairs per line. The $L(\theta)$ curve cannot be steeper than 1 dB per degree, and THETA(NTHETA) must be 90°. Also, the $L(\theta)$ curve must be monotonic. *DEFAULT* : *none* *RANGE* : *angles [0. to 90.]* *losses [0. to 180.]*

***** ENTER LINES 9C–F ONLY IF NBOTM < 0 *****
***** REPEAT LINES 9C–F FOR EACH OF THE –NBOTM REGIONS *****

<u>LINE 9C</u> (F10.2,3I5,5X,F10.2)

COLUMNS	VARIABLE	DESCRIPTION
1–10	RANGE(I)	Range in Nmi for the Ith geo–acoustic bottom region. *DEFAULT* : *none* *RANGE* : *[0. to 25000.]*
11–15	NALPHA(I)	Number of points,up to 30, in the tabulated attenuation vs depth profile. *DEFAULT* : *none* *RANGE* : *[2 to 30]* *RECOMMENDATION*: *as many points as are available*
16–20	NMUD(I)	Number of points, up to 30, in the tabulated bottom sound speed profile. *DEFAULT* : *none* *RANGE* : *[2 to 30]* *RECOMMENDATION*: *as many points as there are available*
21–25	NDENS(I)	Number of points, up to 30, in the tabulated density vs depth profile. NDENS may be set to 1 for a density discontinuity at the water–sediment interface. *DEFAULT* : *0* *RANGE* : *[0 to 30]* *RECOMMENDATION*: *0 or 1*
31–40	CSHEAR(I)	Shear wave velocity in the bottom in feet/sec. *DEFAULT* : *no shear* *RANGE* : *0. to the sound speed at the water-sediment interface* *RECOMMENDATION*: *Shear wave velocity, if known, or zero.*
41–50	RATIO(I)	Sediment–water sound speed ratio, used to determine critical angle for shear wave conversion. *DEFAULT* : *none* *RANGE* : *[.5 to 2.]* *RECOMMENDATION*: *1., unless this quantity is known*

51–60	DENSL(I)	Length in feet of the transition region for density discontinuities. Set DENSL(I) to zero to allow the program to choose this length. *DEFAULT* : *Tappert-computed value* *RANGE* : *[0. to HORRAN/5. (see line 8)]* *RECOMMENDATION*: *0.*

<u>LINE 9D</u> (8F10.2)

<u>COLUMNS</u>	<u>VARIABLE</u>	<u>DESCRIPTION</u>
1–80	(ZALPHA(II),ALPHA(II),II=1,NALPHA(I)	Tabulated attenuation vs depth curve, where ZALPHA is in feet, 0.0 being the water–sediment interface, and ALPHA is the attenuation in dB/ft. *DEFAULT* : *none* *RANGE* : *depths [0. to 100000.]* *alpha's [0. to 300.]*

<u>LINE 9E</u> (8F10.2)

<u>COLUMNS</u>	<u>VARIABLE</u>	<u>DESCRIPTION</u>
1–80	(ZMUD(II),CMUD(II),II=1,NMUD(I))	User–specified sound speed profile in the sediment. Depths (ZMUD) must be in feet, 0.0 being the water–sediment interface, and sound speeds must be in ft/sec. *DEFAULT* : *none* *RANGE* : *depths [0. to 100000.]* *sound speeds [>0. to 50000.]*

<u>LINE 9F</u> (8F10.2)

<u>COLUMNS</u>	<u>VARIABLE</u>	<u>DESCRIPTION</u>
1–80	(ZDENS(II),DENS(II),II=1,NDENS(I))	User–specified density vs depth profile for the sediment. Depths are in feet, density is in gm/cm^3. If only one density is provided, PE assumes a constant density in the sediment, and provides a smooth (hyperbolic tangent function) transition between the water and the sediment. If

more than one density is provided, then PE assumes no discontinuity at the water–sediment interface.

DEFAULT	: *none*
RANGE	: *depths* [*0. to 100000.*]
	densities [*>0. to 10.*]

APPENDIX C
PE Pre–Processor INPUTS

The following pages contain a description of the PE inputs associated with the PE version 2.2 pre-processor. This appendix is taken from the draft of the Software Product Specification for the Parabolic equation model.

PE Version 2.2 Pre-processor INPUT SET

The inputs for the PE pre-processor are identical to those for the standard PE version 2.2 input set with the exception of lines 1 and 9–12. For that reason, lines 1 and 9–12 are described on the following pages, and the user should refer to the previous section of this appendix for input descriptions of other lines.

LINE 1: (A60,A13)

COLUMNS	VARIABLE	DESCRIPTION
1–60	TITLE	Title for this run
61–73	PRFLG	PRFLG must be set to "PRE–PROCESSOR" or to "pre–processor", and subroutine PREP is called immediately to create a PE version 2.2 standard formatted input set from the rest of *this* input set.

LINES 2–8:

SEE APPENDIX II, PART 1

INPUTS FOR LINES 2–8 ARE IDENTICAL TO THOSE IN

THE STANDARD PE VERSION 2.2 INPUT SET

LINE 9 (I5,5X,I5)

COLUMNS	VARIABLE	DESCRIPTION
1–5	NBOTM	NBOTM is the number of bottom regions in range. If NBOTM is greater than zero, the pre-processor will output a loss vs angle input file for PE. If NBOTM is less than zero, the pre-processor will output a geo-acoustic bottom description for PE. NBOTM here has a different meaning from that in the normal (un-pre-processed) PE input set, in that NBOTM determines only the *output* form of the bottom description. The next input on this line, IBLUG, determines the *input* format for bottom loss. *DEFAULT* : *none* *RANGE* : [–9999 *to* –*1*, *1 to* 9999] *RECOMMENDATION*: *as many bottom regions as there are available*
11–15	IBLUG	IBLUG determines the nature of the bottom loss information input to the pre-processor by the user. First, if IBLUG is set to zero, then the input is assumed to "match" the output. For example, if IBLUG is set to zero and NBOTM is +3, then the pre-processor reads loss vs angle tables for three regions in range, and outputs the same to PE. If IBLUG is set to zero and NBOTM is –5, the pre-processor reads and writes out geo-acoustic bottom parameters for 5 bottom regions. Now, if IBLUG is non-zero, the pre-processor will read in BLUG parameters for ABS(NBOTM) bottom regions. The allowable non-zero values for IBLUG are +1 and +2. If IBLUG is +2, system correction factor inputs are read and applied. If IBLUG is +1, system corrections are not made, and only the BLUG parameters are read in and used. The following table lists the various combinations of IBLUG and NBOTM.

VALID INPUT COMBINATIONS FOR LINE 9

NBOTM	IBLUG	NEXT LINE	DESCRIPTION
>0	0	10A–B	Input and output are loss vs angle
<0	0	11A–D	Input and Output are geo-acoustic
>0	2	12A–B	Blug input, loss vs angle output using system correc-

			tion factors
>0	1	13	BLUG input, loss vs angle output, no system correction factors
<0	1	13	BLUG input, geo-acoustic output, no system correction factors
<0	2	NA	Illegal combination

DEFAULT : *No system correction factors*
RANGE : *0 to 2*
RECOMMENDATION : *process-dependent*

***** ENTER LINES 10A AND 10B ONLY IF: *****
NBOTM > 0 AND IBLUG = 0
***** REPEAT LINES 10A–B FOR EACH OF THE NBOTM REGIONS *****

LINE 10A (F10.2,I5)

COLUMNS	VARIABLE	DESCRIPTION
1–10	RANGE(I)	Range in Nmi of the beginning of the Ith bottom region. *DEFAULT* : *none* *RANGE* : *[0.0 to 25000]*
11–15	NTHETA(I)	Number of points in the tabulated loss vs angle curve for this bottom region. This function may contain up to 50 points. *DEFAULT* : *none* *RANGE* : *2 to 50* *RECOMMENDATION*: *As many points as are available, as an increase in the number of points does not cause a significant increase in run time*

LINE 10B (8F10.2)

COLUMNS	VARIABLE	DESCRIPTION
1–80	(THETA(II),LOSS(II),II=1,NTHETA(I))	Tabulated loss vs angle curve for this bottom loss region, four pairs per line. The $L(\theta)$ curve cannot be steeper than 1 dB per degree, and THETA(NTHETA) must be 90°. *DEFAULT* : *none* *RANGE* : *THETA*: *[0 to 90]* : *LOSS*: *[0 to 300.]*

***** ENTER LINES 11A–D ONLY IF: *****
NBOTM < 0 AND IBLUG = 0
***** REPEAT LINES 11A–D FOR EACH OF THE –NBOTM REGIONS *****

<u>LINE 11A</u> (F10.2,3I5,5X,F10.2)

COLUMNS	VARIABLE	DESCRIPTION
1–10	RANGE(I)	Range in Nmi for the Ith geo–acoustic bottom region. *DEFAULT* : *none* *RANGE* : [*0 to 25000.*]
11–15	NALPHA(I)	Number of points,up to 30, in the tabulated attenuation vs depth profile. *DEFAULT* : *none* *RANGE* : *2 to 30* *RECOMMENDATION*: *as many points as are available, as an increase in the number of attenuation points does not cause a significant increase in run time*
16–20	NMUD(I)	Number of points, up to 30, in the tabulated bottom sound speed profile. *DEFAULT* : *none* *RANGE* : *2 to 30* *RECOMMENDATION*: *As many points as are available*
21–25	NDENS(I)	Number of points, up to 30, in the tabulated density vs depth profile. NDENS may be set to 1 for a density discontinuity at the water–sediment interface. *DEFAULT* : *no density profile* *RANGE* : *0 to 30* *RECOMMENDATION*: *0, unless density profile is well-known*
31–40	CSHEAR(I)	Shear wave velocity in the bottom in feet/sec. *DEFAULT* : *no shear wave conversion* *RANGE* : [*0. to sound speed at water-sediment interface*] *RECOMMENDATION*: *0.0*
41–50	RATIO(I)	Sediment–water sound speed ratio, used to deter-

mine critical angle for shear wave conversion.

DEFAULT : *none, if CSHEAR* $\neq 0$
RANGE : *values* ≥ 1
RECOMMENDATION : *1.0*

51–60 DENSL(I) Length in feet of the transition region for density discontinuities. Set DENSL(I) to zero to allow the program to choose this length.

DEFAULT : *program computes transition region thickness*
RANGE : *[0. to 100000.]*
RECOMMENDATION : *0.*

<u>LINE 11B</u> (8F10.2)

<u>COLUMNS</u>	<u>VARIABLE</u>	<u>DESCRIPTION</u>
1–80	(ZALPHA(II),ALPHA(II),II=1,NALPHA(I))	

Tabulated attenuation vs depth curve, where ZALPHA is in feet, 0.0 being the water–sediment interface, and ALPHA is the attenuation in dB/ft.

DEFAULT : *none*
RANGE : *ZALPHA: [0. to 100000.]*
: *ALPHA: [0. to 300.]*

<u>LINE 11C</u> (8F10.2)

<u>COLUMNS</u>	<u>VARIABLE</u>	<u>DESCRIPTION</u>
1–80	(ZMUD(II),CMUD(II),II=1,NMUD(I))	

User–specified sound speed profile in the sediment. Depths (ZMUD) must be in feet, 0.0 being the water–sediment interface, and sound speeds must be in ft/sec.

DEFAULT : *none*
RANGE : *ZMUD: [0. to 10000.]*
CMUD: [>0. to 5000.]

<u>LINE 11D</u> (8F10.2)

<u>COLUMNS</u>	<u>VARIABLE</u>	<u>DESCRIPTION</u>
1–80	(ZDENS(II),DENS(II),II=1,NDENS(I))	

User–specified density vs depth profile for the sediment. Depths are in feet, density is in

gm/cm^3. If only one density is provided, PE assumes a constant density in the sediment, and provides a smooth (hyperbolic tangent function) transition between the water and the sediment. If more than one density is provided, then PE assumes no discontinuity at the water–sediment interface.

DEFAULT	: ***none***
RANGE	: *ZDENS*: [*0. to 100000.*] *DENS*: [*>0. to 10.*]

***** ENTER LINES 12A–B ONLY IF: *****
NBOTM > 0 AND IBLUG = 2
****

<u>LINE 12A</u> (5I5)

<u>COLUMNS</u>	<u>VARIABLE</u>	<u>DESCRIPTION</u>
1–5	IBLFLG	IBLFLG and INTFLG (this line, columns 6–10) determine the relative contributions to bottom loss from BLUG and MGS. IBLFLG determines whether BLUG is used at all, and INTFLG determines whether interpolation is performed between 1000 and 1500 Hz. The following table describes the outcome of the various combinations of IBLFLG and INTFLG: (see table below) *DEFAULT* : *use MGS* *RANGE* : *0 and 1* *RECOMMENDATION*: *1*
6–10	INTFLG	INTFLG is described in the previous paragraph. *DEFAULT* : *don't interpolate* *RANGE* : *0 and 1* *RECOMMENDATION*: *1*
11–15	ISYSFL	Set ISYSFL to 1 to include system correction factors in bottom loss calculations. *DEFAULT* : *no system correction factors* *RANGE* : *0 and 1*
16–20	ITOTFL	Set ITOTFL to 1 to include total energy in bottom loss calculations. *DEFAULT* : *no total energy included* *RANGE* : *0 and 1*
21–25	ITWSPH	Set ITWSPH to 1 for a towed array Set ITWSPH to 2 for a spherical array *DEFAULT* : *none* *RANGE* : *1 and 2*

IBLFLG	INTFLG	<1000Hz	1–1.5kHz	>1.5kHz
0	0	MGS	MGS	MGS
0	1	MGS	MGS	MGS
1	0	BLUG	BLUG	MGS
1	1	BLUG	MIX	MGS

LINE 12B (8F10.2)

COLUMNS	VARIABLE	DESCRIPTION
1–10	BNDWID	Analyzer bandwidth in Hz *DEFAULT* : *none*
11–20	OWNSPD	Own ship speed in knots *DEFAULT* : *none*
21–30	TGTSPD	Target ship speed in knots *DEFAULT* : *none*
31–40	OWNAZI	Own ship azimuth in degrees *DEFAULT* : *end fire*
41–50	TGTAZI	Target ship azimuth in degrees *DEFAULT* : *end fire*
51–60	TALENT	Towed array length in yards. TALENT may be set to zero if this is a spherical array run. *DEFAULT* : *none*
61–70	SALENT	Spherical array length in yards. SALENT may be set to zero for towed array runs. *DEFAULT* : *none*
71–80	SAHITE	Spherical array length in yards. SAHITE may be set to zero if this is a towed array run. *DEFAULT* : *none*

***** ENTER LINES 13A–B ONLY IF IBLUG > 0 *****
***** REPEAT THIS LINE FOR EACH OF THE ABS(NBOTM) REGIONS *****
RECOMMENDATIONS APPLY ONLY IF DATA BASE OR OTHER VALUES ARE NOT AVAILABLE

LINE 13A (8F10.2)

COLUMNS	VARIABLE	DESCRIPTION
1–10	R(I)	Range in Nmi of the Ith BLUG or MGS bottom loss region. *DEFAULT* : *none* *RANGE* : [*0. to 25000.*] *RECOMMENDATION*: *actual ranges*
11–20	T2WAY(I)	Two–way travel time in the sediment in seconds. *DEFAULT* : *none* *RANGE* : [*>0 to 100.*], *but sediments thicker then HORRAN/5 will be truncated for the geo-acoustic bottom* *RECOMMENDATION*: *0.2 in shallow water, 4. in deep water*
21–30	RATIOD(I)	Ratio of sound speed in the sediment to sound speed in the water column at the water-sediment interface. *DEFAULT* : *none* *RANGE* : [*>0 to 10.*] *RECOMMENDATION*: *1.*
31–40	DLD(I)	Thin layer thickness in meters *DEFAULT* : *none* *RANGE* : [*>0 to 1000.*] *RECOMMENDATION*: *0.02*
41–50	RHOSD(I)	Sediment density in gm/cm**3 *DEFAULT* : *none* *RANGE* : [*>0 to 10.*] *RECOMMENDATION*: *1.*
51–60	RHOLD(I)	Thin layer density in gm/cm**3 *DEFAULT* : *none* *RANGE* : [*>0 to 10.*] *RECOMMENDATION*: *use RHOSD(I)*
61–70	GD(I)	Initial gradient in the sediment in 1/sec *DEFAULT* : *gradient of zero*

		RANGE : [$-\infty$ *to* +*100.*] *RECOMMENDATION*: *1.3*
71–80	BETAD(I)	Sound speed profile curvature parameter in the sediment. *DEFAULT* : *0.0* *RECOMMENDATION*: *–0.5*

LINE 13B (3F10.2,I5,5X,F10.2)

COLUMNS	VARIABLE	DESCRIPTION
1–10	FKZD(I)	Attenuation at the top of the sediment layer in dB/m/kHz *DEFAULT* : *0.0* *RANGE* : [*0. to 300.*] *RECOMMENDATION*: *0.01*
11–20	FKZP(I)	Attenuation gradient in the sediment in dB/m/kHz/m *DEFAULT* : *0.0* *RANGE* : [*0. to 10.*] *RECOMMENDATION*: *0.00001*
21–30	BRFLD(I)	Bottom reflection coefficient (the basement "loss" is –20log(BRFLD)) *DEFAULT* : *none* *RANGE* : [>*0 to 1*] *RECOMMENDATION*: *0.5*
31–35	MGS(I)	MGS province number *DEFAULT* : *1* *RANGE* : *1 to 9*
41–50	FEXP(I)	Exponential term applied to frequency when computing attenuation in the sediment. The sediment attenuation in dB/meter at any depth Z in the sediment is defined as $(FKZ(I)+Z*FKZP(I))*F^{FEXP(I)}$ where F is the frequency in KHz. *DEFAULT* : *1.0* *RANGE* : [*–100. to 100.*] *RECOMMENDATION*: *1.0, until the "10th" BLUG parameter is availabel in data bases*

APPENDIX D
SAMPLE INPUT AND OUTPUT FILES

TABLE D-1
PE PRE-PROCESSOR INPUT FILE - BLUG TO LOSS VS. ANGLE

```
BLUG PARAMETERS TO LOSS V. ANGLE TEST CASE                    PRE-PROCESSOR
    6     0     1    60     0
30.           25.         20.0
12500.      80.          0.          0.          16000.      70.         10.
20.         50.          100.        200.        300.        400.
    1
0.                2
0.          1520.        3685.       1520.
6000.
    2           1
0.00        1.50        .999        1.00        1.15        1.15        .6          -.5
.018        .0001       .500
20.00       .100        1.03        1.00        1.00        1.00        .9          -.5
.200        .00001      .500
```

TABLE D-2
PRE-PROCESSOR CONVERTED RESULTS - LOSS VS. ANGLE

BLUG PARAMETERS TO LOSS V. ANGLE TEST CASE

```
    6      0     1    60     0      0      0      0      0      0      0      0     000000
30.00     25.00      20.00      0.00       0.00       0.00       0.00
12500.    80.00      0.00       0.00       16000.00   70.00      10.00
20.00     50.00      100.00     200.00     300.00     400.00
    1     0
0.0            2
0.00      1520.00    3685.00    1520.00
6000.00
    2
0.00         12
0.00      0.00       1.00       0.82       13.00      0.82       14.00      0.85
15.00     0.97       16.00      1.10       17.00      1.26       18.00      1.44
19.00     1.66       20.00      1.92       21.00      2.24       90.00      11.55
20.00        20
0.00      0.00       1.00       0.12       2.00       0.23       3.00       0.35
4.00      0.48       5.00       0.62       6.00       0.76       7.00       0.93
8.00      1.12       9.00       1.34       10.00      1.63       11.00      2.01
12.00     2.57       13.00      3.52       14.00      4.4[illegible]  15.00      5.21
18.00     5.21       19.00      6.03       21.00      7.93       90.00      9.45
```

TABLE D-3
PE PRE-PROCESSOR INPUT FILE - BLUG TO GEOACOUSTIC PARAMETERS

```
 BLUG PARAMETERS TO GEO-ACOUSTIC BOTTOM TEST CASE              PRE-PROCESSOR
     6     0     1    60     0
30.         25.         4.5
12500.      40.         0.          0.          11500.      70.         10.
20.         50.         100.        200.        300.        400.
     1
0.               2
0.          1520.       3685.       1520.
6000.
    -2           1
0.00        1.50        .999        1.00        1.15        1.15        .6          -.5
.018        .0001       .500
20.00       .100        1.03        1.00        1.00        1.00        .9          -.5
.200        .00001      .500
```

TABLE D-4
PRE-PROCESSOR CONVERTED RESULTS - GEOACOUSTIC PARAMETERS

BLUG PARAMETERS TO GEO-ACOUSTIC BOTTOM TEST CASE

```
   6    0     1    60
30.00   25.00     4.50      0.00      0.00      0.00      0.00
12500.  40.00     0.00      0.00      11500.    70.00     10.00
20.00   50.00     100.00    200.00    300.00    400.00
   1    0
0.0           2
0.00    1520.00   3685.00   1520.00
6000.00
  -2
0.0           2    21     1 0.0       1.0       0.0
0.00    .0001372  4438.41   .0011680
0.00    5000.     221.92    5129.78   443.84    5253.46   665.76    5371.81
887.68  5485.47   1109.60   5594.97   1331.52   5700.71   1553.44   5803.08
1775.36 5902.37   1997.29   5998.84   2219.21   6092.72   2441.13   6184.21
2663.05 6273.48   2884.97   6360.69   3106.89   6445.97   3328.81   6529.44
3550.73 6611.23   3772.65   6691.41   3994.57   6770.10   4216.49   6847.35
4438.41 6923.26
0.0     1.15
20.00         3    22     1 0.0       1.03      0.0
0.00    0.001524  251.52    .0015298  1200.00   0.0015298
0.00    5000.     12.58     5011.29   25.15     5022.54   37.73     5033.73
50.30   5044.87   62.88     5055.97   75.46     5067.01   88.03     5078.01
100.61  5088.96   113.18    5099.87   125.76    5110.73   138.34    5121.55
150.91  5132.32   163.49    5143.05   176.06    5153.73   188.64    5164.37
201.22  5174.97   213.79    5185.53   226.37    5196.04   238.94    5206.52
251.52  5216.95   1200.00   5216.95
0.00    1.00
```

TABLE D-5
INPUT FILE FOR USER-SELECTED TRANSITION LENGTH

```
DENSITY DISCONTINUITY TEST CASE
    5     0     1    60     0     0     0     0     0     0     0     0      00000
30.00     25.00     20.00     0.00      0.00      0.00      0.00      0.00
12000.    80.00     0.00      0.00      13200.    70.00     10.00     1200.
50.00     100.00    200.00    300.00    400.00
    1     0
0.0           2
0.00      1520.00   3657.60   1520.00
6000.00
   -2
0.0           2     2     2   0.0       1.00      0.0
0.00      .0001372  1200.     .0001400
0.00      5000.00   1200.00   5000.00
0.00      1.15      1200.00   1.20
20.           2     2     1   0.00      1.00      20.00
0.00      .0001372  1200.00   0.0001400
0.00      5000.00   1200.00   5000.00
0.00      1.15
```

TABLE D-6
ABBREVIATED PE OUTPUT FILE FOR USER SELECTED TRANSITION LENGTH

```
PE VERSION 2.1 - March 19, 1987. RUN DATE11/03/87RUN TIME15:11:26.18

DENSITY DISCONTINUITY TEST CASE


INPUT DATA --
PARAMETERS -    5    0    1   60    0    0    0    0    0    0    0    0
FLAGS        00000

   5 OUTPUT DEPTHS
THE BOTTOM IS FLAT.

SPHERICAL EARTH CORRECTION APPLIED


DENSITY DISCONTINUITY TEST CASE



PROFILE AT RANGE =          0.00 NM
DEPTH(FT)     C(Z)(FT/S)      DEPTH(M)      C(Z)(M/S)
       0.        4986.88            0.        1520.00
   12000.        4986.88         3658.        1520.00


MAXIMUM DEPTH   = 12000.0 FT
MAXIMUM RANGE   =   80.00 NM
SOURCE DEPTH    =    30.0 FT
FREQUENCY       =    25.0 HZ
BEAM WIDTH      =    20.0 DEGREES
OUTPUT DEPTHS IN FT
               1.    50.0
               2.   100.0
               3.   200.0
               4.   300.0
               5.   400.0
VARIABLE STEP SIZE RUN.
```

CMOD PHASE CORRECTION USED

TABLE D-6 (CONTINUED)

CO (REFERENCE SOUND SPEED) = 4986.829 FT/SEC = 1519.985 M/S

SELECTED TRANSFORM SIZE = 2** 7
EFFECTIVE BEAMWIDTH (DEG) = 20.0
MESH SPACING = 218.71 FT

2 BOTTOM REGIONS WITH 1200.00 FT SEDIMENT THICKNESS

REGION 1 BEGINNING AT RANGE 0.000 NM

BOTTOM ATTENUATION PROFILE

DEPTH IN BOTTOM (FT)	ALPHA (DB/FT)
0.00	0.000137
1200.00	0.000140

BOTTOM DENSITY PROFILE

DEPTH IN BOTTOM (FT)	DENSITY (GM/CM**3)
0.00	1.150
1200.00	1.200

TABLE D-6 (CONTINUED)

BOTTOM SOUND SPEED PROFILE MODIFIED FOR DENSITY

DEPTH IN BOTTOM (FT)	SPEED (FT/SEC)
0.00	5000.00
48.00	5000.00
96.00	5000.00
144.00	5000.00
192.00	5000.00
240.00	5000.00
288.00	5000.00
336.00	5000.00
384.00	5000.00
432.00	5000.00
480.00	5000.00
528.00	5000.00
576.00	5000.00
624.00	5000.00
672.00	5000.00
720.00	5000.00
768.00	5000.00
816.00	5000.00
864.00	5000.00
912.00	5000.00
960.00	5000.00
1008.00	5000.00
1056.00	5000.00
1104.00	5000.00
1152.00	5000.00
1200.00	5000.00

TABLE D-6 (CONTINUED)

REGION 2 BEGINNING AT RANGE 20.000 NM

BOTTOM ATTENUATION PROFILE

DEPTH IN BOTTOM (FT)	ALPHA (DB/FT)
0.00	0.000137
1200.00	0.000140

BOTTOM DENSITY PROFILE

DEPTH IN BOTTOM (FT)	DENSITY (GM/CM**3)
0.00	1.150

USER-SUPPLIED DENSITY TRANSITION REGION LENGTH = 20.00 FT

BOTTOM SOUND SPEED PROFILE MODIFIED FOR DENSITY

DEPTH IN BOTTOM (FT)	SPEED (FT/SEC)
0.00	5000.00
1.05	4864.95
2.11	4876.41
3.16	4890.20
4.21	4906.23
5.26	4924.35
6.32	4944.31
7.37	4965.77
8.42	4988.34
9.47	5011.57
10.53	5034.96
11.58	5057.98
12.63	5080.11
13.68	5100.87
14.74	5119.82
15.79	5136.60
16.84	5150.94
17.89	5162.66
18.95	5171.69
20.00	5178.05
1200.00	5000.00

TABLE D-7
INPUT FILE FOR PROGRAM-SELECTED TRANSITION LENGTH

```
 DENSITY DISCONTINUITY TEST CASE
    5    0    1   60    0    0    0    0    0    0    0    0    00000
    30.00     25.00     20.00      0.00      0.00      0.00      0.00
0.00
 12000.00     80.00      0.00      0.00  13200.00     70.00     10.00
1200.00
    50.00    100.00    200.00    300.00    400.00
    1    0
     0.00    2
     0.00   1520.00   3657.60   1520.00
  6000.00
   -2
     0.00    2    2    2          0.00      1.00      0.00
     0.00 0.0001372   1200.00 0.0001400
     0.00   5000.00   1200.00   5000.00
     0.00      1.15   1200.00   1.20
    20.00    2    2    1          0.00      1.00      0.00
     0.00 0.0001372   1200.00 0.0001400
     0.00   5000.00   1200.00   5000.00
     0.00      1.15
```

TABLE D-8

ABBREVIATED OUTPUT FILE FOR PROGRAM-SELECTED TRANSITION LENGTH

```
PE VERSION 2.1 - March 19, 1987. RUN DATE11/10/87RUN TIME14:52:38.34

DENSITY DISCONTINUITY TEST CASE


INPUT DATA --
PARAMETERS -    5    0    1   60    0    0    0    0    0    0    0    0
FLAGS         00000

   5 OUTPUT DEPTHS
THE BOTTOM IS FLAT.

SPHERICAL EARTH CORRECTION APPLIED



DENSITY DISCONTINUITY TEST CASE



PROFILE AT RANGE =          0.00 NM
DEPTH(FT)      C(Z)(FT/S)       DEPTH(M)       C(Z)(M/S)
      0.          4986.88             0.          1520.00
  12000.          4986.88          3658.          1520.00


MAXIMUM DEPTH   = 12000.0 FT
MAXIMUM RANGE   =    80.00 NM
SOURCE DEPTH    =     30.0 FT
FREQUENCY       =     25.0 HZ
BEAM WIDTH      =     20.0 DEGREES
OUTPUT DEPTHS IN FT
                 1.   50.0
                 2.  100.0
                 3.  200.0
                 4.  300.0
                 5.  400.0
VARIABLE STEP SIZE RUN.
```

CMOD PHASE CORRECTION USED

TABLE D-8 (CONTINUED)

CO (REFERENCE SOUND SPEED) = 4986.829 FT/SEC = 1519.985 M/S

SELECTED TRANSFORM SIZE = 2** 7
EFFECTIVE BEAMWIDTH (DEG) = 20.0
MESH SPACING = 218.71 FT

2 BOTTOM REGIONS WITH 1200.00 FT SEDIMENT THICKNESS

REGION 1 BEGINNING AT RANGE 0.000 NM

BOTTOM ATTENUATION PROFILE

DEPTH IN BOTTOM (FT)	ALPHA (DB/FT)
0.00	0.000137
1200.00	0.000140

BOTTOM DENSITY PROFILE

DEPTH IN BOTTOM (FT)	DENSITY (GM/CM**3)
0.00	1.150
1200.00	1.200

TABLE D-8 (CONTINUED)

BOTTOM SOUND SPEED PROFILE MODIFIED FOR DENSITY

DEPTH IN BOTTOM (FT)	SPEED (FT/SEC)
0.00	5000.00
48.00	5000.00
96.00	5000.00
144.00	5000.00
192.00	5000.00
240.00	5000.00
288.00	5000.00
336.00	5000.00
384.00	5000.00
432.00	5000.00
480.00	5000.00
528.00	5000.00
576.00	5000.00
624.00	5000.00
672.00	5000.00
720.00	5000.00
768.00	5000.00
816.00	5000.00
864.00	5000.00
912.00	5000.00
960.00	5000.00
1008.00	5000.00
1056.00	5000.00
1104.00	5000.00
1152.00	5000.00
1200.00	5000.00

TABLE D-8 (CONTINUED)

REGION 2 BEGINNING AT RANGE 20.000 NM

BOTTOM ATTENUATION PROFILE

DEPTH IN BOTTOM (FT)	ALPHA (DB/FT)
0.00	0.000137
1200.00	0.000140

BOTTOM DENSITY PROFILE

DEPTH IN BOTTOM (FT)	DENSITY (GM/CM**3)
0.00	1.150

BOTTOM SOUND SPEED PROFILE MODIFIED FOR DENSITY

DEPTH IN BOTTOM (FT)	SPEED (FT/SEC)
0.00	5000.00
3.34	4986.10
6.68	4987.32
10.03	4988.77
13.37	4990.46
16.71	4992.34
20.05	4994.39
23.39	4996.57
26.73	4998.84
30.08	5001.14
33.42	5003.44
36.76	5005.66
40.10	5007.78
43.44	5009.74
46.79	5011.51
50.13	5013.07
53.47	5014.39
56.81	5015.46
60.15	5016.27
63.49	5016.85
1200.00	5000.00

APPENDIX E

PE VERSION 2.2 OUTPUTS

PE creates several output files for user access. The most obvious of these is the standard printed PE output file assigned to unit 6, however if the user wishes to manipulate actual transmission loss values or complex pressures, files with more convenient format are provided. These output files are:

UNIT #	FILE NAME	GENERIC NAME/DESCRIPTION
6	text output	PE OUTPUT FILE / Formatted PE output file.
3	for003.dat	PE CONTOUR FILE / Transmission loss vs range at up to 120 equally spaced output depths, used by the PE contour plotter.
41	for041.dat	PE TRANSMISSION LOSS FILE / Transmission loss vs range at up to 20 user-specified output depths, for use by transmission loss plotting routines.
44	for044.dat	PE COMPLEX PRESSURE FILE / Complex pressure vs range and depth at user–specified ranges and depths, for various uses, including the beamformer program, which produces level vs angle.

In the file descriptions, all integer and floating point variables are assumed to be four bytes long. The exception to this rule is the version of PE V2.2 written for the PC, in which all integers are two bytes long by default.

PE OUTPUT FILE

The PE output file is a formatted (ASCII) file containing both the user inputs and standard and optional program outputs. The standard outputs are the computed depth mesh size and spacing, the effective beamwidth and any computed bottom properties. The optional outputs consist of a line printer contour plot and printed transmission loss vs range at the output depths. Inputs controlling the line printer contour plot are:

NPLT	See line 2, columns 16–20 of the PE input set.
IDENSE	line 2, column 67
CD1	line 3, columns 31–40
CD2	line 3, columns 41–50
CLMIN	line 3, columns 51–60
DCL	line 3, columns 61–70, and
BCORR	line 3, columns 71–80.

Transmission loss versus range printout is controlled by the input variable IPRNT, line 2 columns 11–15.

PE CONTOUR FILE

The PE contour file is written at all ranges and depths chosen for the line printer field plot. Setting NPLT to a negative number causes the PE contour file to be generated. This is an unformatted file with the following record format:

Record 1: NNN,(CD(K),K=1,NNN),WHEN,F,ZS

where	NNN	is the number of depths at which transmission loss is provided in subsequent records. NNN is commonly NPLT−1, since there is no use in saving the infinite transmission loss at the surface (although the surface is included in the line printer contour plot).
	CD	is the array of depths in feet at which transmission loss is provided.
	WHEN	is a character*9 variable, set to the date on the PC and the VAX, set to nonsense on the HP9020 since we don't know how to find the date from inside our FORTRAN program.
	F	is frequency in Hz.
	ZS	is the input (stationary point) depth in feet.

Records 2–EOF: RNM,ZBOT,(CTL(K),K=1,NNN)

where	RNM	is the range in nautical miles.
	ZBOT	is the bottom depth in feet
	CTL	is the array of transmission losses at the contour depths from record 1.

PE TRANSMISSION LOSS FILE

The transmission loss file contains transmission loss vs range at the ND user–specified output depths from line 5 of the PE input set. This file, like the contour file, is an unformatted file with a header and many data records:

RECORD 1: TITLE,WHEN,F,ZS,NR,ND,(D(I),I=1,ND),AVER,THMIN,THMAX,NPROF, NBOTM

where	TITLE	is an 80 byte integer array containing the title of the PE run.
	WHEN	is a character*9 variable containing the date of the PE run (except on the HP9020, in which case WHEN contains junk).
	F	is the frequency in Hz.
	ZS	is the input (stationary point) depth in feet.
	NR	is zero. Some models insert the number of range points here, but as PE has a variable range step, the number of range points is unknown at the time this header record is written.
	ND	is the number of output depths.
	D	is the array of output depths in feet.
	AVER	is the amount of range averaging reflected in the transmission loss. The transmission loss file created by PE will always have AVER set to zero. AVER will be changed on output from the range averaging program.
	THMIN	is the minimum (most negative) angle in a directed source. If the user has not specified a directed source, then THMIN and THMAX will be set to zero.
	THMAX	is the maximum angle in the directed source.
	NPROF	is the number of sound speed profiles input by the user. If NPROF is zero, then the profiles were read in from the output of a CFIELD run.
	NBOTM	is the number of bottom loss regions. As in the PE input set, if NBOTM is greater than zero, a loss vs angle function is used, and if NBOTM is less than zero, a geo-acoustic bottom is used.

RECORDS 2–EOF: RNM,(TL(I),I=1,ND)

where	RNM	is the range in nautical miles.
	TL	is the array of transmission loss values in dB re 1 yard.

PE COMPLEX PRESSURE FILE

While the complex pressure file was originally supplied for input to the PE beamformer (which supplies level vs angle and range), it clearly can be used for other purposes. The complex pressures are output as PR and PI, where the units for PR and PI are such that

$$TL = 10 \log_{10}((PR^2+PI^2)*RR)$$

where RR, called the reciprocal range, is 9/range in feet.

The contents of the complex pressure file are controlled by NRBEAM on line 2, and the inputs on line 2F. Basically, the complex pressures are saved at all mesh points between depths Z44MIN and Z44MAX, and at user–specified ranges. This is an unformatted file.

RECORD 1: TITLE,WHEN,F,ZS,NR,ND,(D(I),I=1,ND),AVER,THMIN,THMAX,NPROF, NBOTM

where		This header record is identical to the first record of the PE transmission loss file.

RECORD 2: DZ,NP44,N441,N442,C0

where	DZ	is the mesh spacing in feet.
	NP44	is the number of depths at which complex pressure is provided.
	N441	is the index of the shallowest mesh point at which complex pressure is provided. The depth in feet corresponding to N441 is N441*DZ. This depth is less than or equal to Z44MIN.
	N442	is the index of the deepest mesh point at which complex pressure is provided. The depth in feet corresponding to N442 is N442*DZ. This depth is greater than or equal to Z44MAX.
	C0	is the PE reference sound speed. The PE beamformer assumes a constant sound speed of C0 along its vertical arrays.

RECORDS 3–EOF: RNM,(PR(I),PI(I),I=N441,N442)

where	RNM	is the range in nautical miles.
	PR	is the real part of the complex pressure.
	PI	is the imaginary part of the PE complex pressure. It should be noted that PE propagates a complex conjugate field, and this should be taken into account when using the complex pressures on this file. The PE beamformer takes this into account by implicitly negating the imaginary part of the PE complex pressure.

APPENDIX F

AN ANALYSIS OF RANGE STEP AND MESH SPACING REQUIREMENTS FOR THE SPLIT-STEP PARABOLIC EQUATION ALGORITHM

By: John S. Hanna

AN ANALYSIS OF RANGE STEP AND MESH SPACING REQUIREMENTS FOR THE SPLIT-STEP PARABOLIC EQUATION ALGORITHM

I. INTRODUCTION

This paper reviews the errors induced in the solution of the parabolic equation[1] by the split-step algorithm[2] and the implications they have for depth-mesh and range-step selections. To place these errors in context, the several steps required to go from the parabolic equation to its solution using the split-step algorithm are considered. This context is presented below and is used to develop the approach taken in analyzing the errors. The error bounds are developed and quantified in Section II and a summary presented in Section III. Section IV contains example calculations illustrating the results of Section II.

To develop the estimates of errors in the split-step algorithm, it is convenient to follow the approach of Thompson and Chapman[3] who show that the parabolic equation for the radially outgoing field is

$$Pu = ik_o Qu \tag{1}$$

where

$$P = \frac{\partial}{\partial r},$$

$$Q = (1 + \varepsilon + \mu)^{1/2},$$

$$\varepsilon = n^2 - 1,$$

$$\mu = \frac{1}{k_o^2}\frac{\partial^2}{\partial z^2},$$

$$n = \frac{c_o}{c(r,z)} ,$$

$$k_o = \frac{\omega}{c_o} ,$$

and u(r,z) is related to the desired pressure field by

$$p(r,z) = \frac{1}{\sqrt{r}} u(r,z) .$$

Of course, solutions to the elliptic wave equation are desired and the solutions to Equation 1 only provide approximations. In what follows, we are not concerned with these approximations, but only with the errors accumulated in solving Equation 1.

The errors inherent in the split-step solution to Equation 1 can be divided into three categories. The first category corresponds to approximating the square root operator, Q, and some candidates are discussed in Reference 3. Since the parabolic equation code with which this paper is concerned is based on the form of Q suggested by Thompson and Chapman, only that representation is considered here. Specifically, they recommend

$$\hat{Q} = (1+\mu)^{1/2} + [(1+\varepsilon)^{1/2} - 1] \qquad (2)$$

which leads to

$$P\hat{u} = ik_o\hat{Q}\hat{u} \qquad (3)$$

and $\hat{u}$ explicitly reminds us that Equation 3 is an approximation to Equation 1.

Making the substitution $\hat{u}(r,z) = \hat{\psi}(r,z) \exp(ik_o r)$, they show, using Equation 2, that

$$P\hat{\psi} = ik_o\mu\ [(1+\mu)^{1/2} + 1]^{-1}\ \hat{\psi} + ik_o(n-1)\ \hat{\psi}.$$

Using the notation

$$A = [(k_o^2 + \frac{\partial^2}{\partial z^2})^{1/2} + k_o]^{-1} \frac{\partial^2}{\partial z^2},$$

$$B = k_o[n(r,z)-1]$$

yields

$$P\ \hat{\psi} = i(A+B)\ \hat{\psi} \qquad (4)$$

whose formal solution is

$$\hat{\psi}(r+\Delta r,z) = \exp[i(A+B)\Delta r]\ \hat{\psi}(r,z). \qquad (5)$$

So, the first category of errors contains the differences between $\hat{u}(r,z)$ and $u(r,z)$ created by replacing Q by $\hat{Q}$. These errors are the subject of Reference 3 and are not considered further here.

The second category of errors corresponds to a rearrangement of the exponential operator in Equation 5 for the purpose of implementing the split-step algorithm. This algorithm replaces the differential operator form of A by its multiplicative equivalent in the Fourier transform space of k_z. This requires factoring the exponential operator which introduces an error since A and B do not commute. The factored form used in the SAIC code at this time is

$$\tilde{\psi}(r+\Delta r,z) = \exp(iB\Delta r)\ \exp(iA\Delta r)\ \tilde{\psi}(r,z) \qquad (5)$$

where $\tilde{\psi}$ is now an approximation to $\hat{\psi}$. The second category of errors, then, contains the differences between $\tilde{\psi}(r,z)$ and

$\hat{\psi}(r,z)$. However, the difference between $\exp(iA+iB)$ and $\exp(iB)\cdot\exp(iA)$ can be expanded in terms of Δr and the commutators of A and B which leads to errors that grow with Δr. These errors are also not the subject of the present analysis.

The subject of this paper is the third category of errors involved with the solution to Equation 5 using the split-step Fourier algorithm. In particular, if we let F denote the transform from z to k_z and F^{-1} the inverse transform, the split-step solution to Equation 5 is

$$\tilde{\psi}(r+\Delta r,z) = \exp[ik_o\Delta r(n(r,z)-1)] \times F^{-1}(\tilde{\psi}(r,k_z)) \quad (6)$$

where

$$\tilde{\psi}(r,k_z) = \exp\{-ik_z^2\Delta r[(k_o^2-k_z^2)^{1/2}+k_o]^{-1}\}\cdot F(\tilde{\psi}(r,z)).$$

To keep the notation compact, return to the form

$$\tilde{\psi}(r+\Delta r,z) = \exp(iB\Delta r)F^{-1}[\exp(iA\Delta r)\ F(\tilde{\psi}(r,z))] \quad (7)$$

where A is understood to be the k_z-space form of the operator. Since the Fourier transforms, which are continuous in Equation 7, are accomplished in the split-step algorithm by discrete transforms, we seek to quantify the constraints on the depth sampling, Δz, and the range step, Δr, which are imposed by the need to satisfy the usual sampling theorem requirements. In principle, if Equation 7 were solved using continuous transforms over all z and k_z space, the only constraint on Δr would be imposed by the rate at which B(r,z) changes with r. As we will see below, solving Equation 7 using discrete transforms imposes constraints on Δr which are related to the choice of depth mesh.

To summarize, the errors in solving Equation 1 are categorized as they follow from the steps listed below:

(1) Replacement of the operator Q by $\hat{Q}$.

(2) Replacement of the factor exp(iA+iB) by the factor exp(iB) · exp(iA).

(3) Replacement of the continuous Fourier transform in Equation 7 by the discrete transform.

The remainder of this paper considers the requirements on depth and range sampling imposed by the third category of errors.

II. ERROR ESTIMATES AND SAMPLING REQUIREMENTS

A. Technical Approach

If we set aside consideration of the range dependence of $B = k_o[n(r,z)-1]$ and the constraint it might impose on Δr (which is likely to be much less restrictive than the limits discussed below), consideration of Equation 7 shows the need for the following:

> The spatial sampling of $\tilde{\psi}(r,z)$ must be adequate for (1) the wavenumber spectrum of $\tilde{\psi}(r,z)$, (2) the wavenumber spectrum of $\exp(iB\Delta r)$ and (3) the depth dependence of $F^{-1}[\exp(iA\Delta r)]$.

The approach taken here is to establish the requirement on depth sampling and then determine the limits on Δr which must be observed to insure that the two exponential operators are properly sampled.

B. Requirement Imposed by $\tilde{\psi}$

In the SAIC parabolic equation model, the initial field, $\tilde{\psi}(o,z)$, is determined by its wavenumber transform. Specifically, a band-limited description (i.e., $|k_z| \leq k_o \sin\theta_{max}$) of $F[\tilde{\psi}(o,z)]$ is set up which satisfies the pressure-release boundary condition at the ocean surface and corresponds to the user-specified initial field depth and maximum angle (determined by $k_1 = k_o \sin\theta_{max}$). The maximum wavenumber, k_1, determines the required depth sampling

$$\Delta z_1 = \frac{2\pi}{2k_1} = \frac{\pi}{k_1} \qquad (8)$$

and the maximum depth for the problem of interest determines the size of the transform, $N = z_{max}/\Delta z_1$. As the solution is advanced by the split-step algorithm, a low-pass filter is applied in both the z and k_z spaces to prevent aliasing. Thus, the algorithm insures that the initial selection of the depth sampling will remain adequate to represent $\tilde{\psi}(r,z)$.

C. Requirements Imposed by $\exp(iB_\Delta r)$

The depth sampling selected by consideration of $\tilde{\psi}$ will be adequate to represent $\exp(iB_\Delta r)$ provided the wavenumber spectrum of this function does not have significant contributions at wavenumber exceeding k_1. To determine this, consider the spectrum of the operator given by

$$\phi(k) = \int e^{ikz} \cdot e^{i\Delta r k_o(n-1)} dz .$$

We use a stationary phase argument to determine that portion of $n(r,z)$ which contributes to the spectrum at a particular wavenumber, k. Stationary phase requires

$$\frac{\partial}{\partial z} (kz + \Delta r k_o(n-1) = 0$$

or

$$k = -\Delta r k_o \frac{\partial n}{\partial z}$$

$$= \Delta r k_o \frac{c'(z) \cdot c_o}{c^2(z)} \quad (9)$$

where the variable r has been suppressed as an explicit argument of c and the prime denotes differentiation with depth. Equation 9 suggests that the highest wavenumber component of $\exp(iB\Delta r)$ will be determined by the largest value of c'/c^2. Actually, the implication goes somewhat further because the Fourier integral over the z-neighborhood for which c'/c^2 is nearly constant produces a spectral peak whose k-width is inversely proportional to the width of this z-neighborhood. To see this explicitly, suppose c'/c^2 is approximately constant from z_1 to z_2 and let the value of k from Equation 8 be

$$k_m = \Delta r k_o \frac{c'(z) \cdot c_o}{c^2(z)} \quad .$$

Then the spectral peak is given by

$$\phi(k) = \int_{z_1}^{z_2} e^{ikz-ik_m z}\, dz$$

$$= e^{i(k-k_m)\,\delta/2} \cdot \mathrm{sinc}((k-k_m)\,\delta/2)$$

where $\delta = z_2 - z_1$. This represents a peak centered at $k=k_m$ whose width (at the first zero) is given by

$$(k-k_m)\,\delta/2 = \pi$$

from which

$$k = \frac{2\pi}{\delta} + k_m .$$

c'(z) gradient

In order to keep the spectral peak at k_m from being aliased when the transition from continuous to discrete transforms is made, the maximum wavenumber must satisfy

$$k_{max} \geq \frac{2\pi}{\delta} + k_m = \frac{2\pi}{\delta} + \Delta r k_o \left(\frac{c'(z)c_o}{c(z)^2}\right)_{max} .$$

The second term on the right in this inequality can be reduced arbitrarily by the selection of Δr; however, the first term is a constant determined by the depth interval over which the expression $c'(z)c_o/c(z)^2$ is constant. Although the relative sizes of these two terms could be chosen arbitrarily (provided k_{max} was adjusted appropriately), it seems reasonable to let them be of equal magnitude which results in the requirements

$$k_{max} \geq 2 \cdot \frac{2\pi}{\delta} \tag{10}$$

and

$$\Delta r k_o \left(\frac{c'(z)c_o}{c(z)^2}\right)_{max} \leq \frac{2\pi}{\delta}$$

or

$$\Delta r \leq \frac{\lambda}{\delta\left(\frac{c'(z)c_o}{c(z)^2}\right)_{max}} \tag{11}$$

Since k_{max} is related to the spacing for the depth mesh by

$$\Delta z_2 = \frac{2\pi}{2k_{max}} = \frac{\pi}{k_{max}}$$

we can rewrite Equation 10 as

$$\delta \geq \frac{4\pi}{k_{max}} = 4\Delta z$$

or

$$\Delta z_2 \leq \delta/4 \ . \qquad (12)$$

The selected depth sampling must be the lesser of Δz_2 in Equation 12 and Δz_1 in Equation 8 of Section II.B.

D. Requirements Imposed by $\exp(i A_\Delta r)$

Using the form of this operator in wavenumber space

$$G(k_z) = \exp\left(-i\Delta r \frac{k_z^2}{(k_o^2 - k^2)^{1/2} + k_o}\right) = \exp\left(-i\overline{\Delta r} \frac{\kappa^2}{(1-\kappa^2)^{1/2} + 1}\right)$$

where

$$\overline{\Delta r} = k_o \Delta r ,$$

and

$$\kappa = \frac{k_z}{k_o} ,$$

it is convenient to view G as a sinusoidal function whose instantaneous frequency varies with κ. Defining this frequency as

$$\mathfrak{z} = \frac{d}{d\kappa}\left(\overline{\Delta r} \frac{\kappa^2}{(1-\kappa^2)^{1/2} + 1}\right)$$

and carrying out the derivative yields

$$\mathfrak{z} = \overline{\Delta r}\, \frac{\kappa}{(1-\kappa^2)^{1/2} + 1} \left[2 + \frac{\kappa^2}{(1-\kappa^2)^{1/2}((1-\kappa^2)^{1/2} + 1)}\right]$$

If we let $k_z = k_o \cdot \sin\theta$, a little algebra results in

$$\mathfrak{z} = \overline{\Delta r} \cdot \tan\theta . \tag{13}$$

Clearly, this frequency will be greatest for the maximum value of k_z or, equivalently, θ. Given the maximum depth, z_{max}, for a particular case, the k_z samples will have a spacing

$$\Delta k_z = \frac{\pi}{z_{max}}$$

or

$$\Delta\kappa = \frac{\lambda}{2 \cdot z_{max}} . \tag{14}$$

For $\Delta\kappa$ to sample $G(k_z)$ adequately at its highest instantaneous frequency, we must have

$$\Delta\kappa \cdot \mathfrak{z}_{max} \leq \pi \tag{15}$$

which is just the Nyquist requirement. Consequently, Equation 15 requires (using Equations 13 and 14)

$$\frac{\lambda}{2 \cdot z_{max}} \cdot \overline{\Delta r} \cdot \tan\theta_{max} \leq \pi$$

or

$$\frac{\lambda}{2 \cdot z_{max}} \cdot \frac{2\pi \Delta r}{\lambda} \cdot \tan\theta_{max} \leq \pi$$

Thus, the range step must satisfy

$$\Delta r \leq \frac{z_{max}}{\tan\theta_{max}} \tag{16}$$

which is independent of the acoustic frequency.

Note that the requirements on k_{max} from Sectio· II.C may result in $k_{max} > k_o \cdot \sin\theta_{max}$ where θ_{max} is the largest equivalent angle for the propagating energy (as used in Section II.B). It is consistent here to be concerned with $G(k_z)$ only up to $k_o \sin\theta_{max}$ because it multiplies the wave-number spectrum of $\tilde{\psi}$ which is never allowed to have components at wavenumbers exceeding $k_o \sin\theta_{max}$. As mentioned earlier, this is insured by the use of a low-pass filter in k_z-space at each range step.

III. SUMMARY OF RANGE STEP AND MESH SPACING REQUIREMENTS

For the split-step algorithm to sample the acoustic field in depth and range with an accuracy that insures proper behavior of the Fourier transforms we have the following constraints:

Depth Mesh

The depth mesh must be minimum of

$$\Delta z_1 = \frac{\pi}{k_o \sin\theta_{max}}$$

and

$$\Delta z_2 \leq \delta/4$$

where δ is the depth interval corresponding to

$$\left(\frac{c'(z)c_o}{c(z)^2}\right)_{max} = \text{constant}$$

units $\frac{\text{ft/sec}}{\text{ft}} \frac{\text{ft/s}}{\text{ft}^2/\text{sec}^2} = \frac{1}{\text{ft}}$

Range Step

Given the depth mesh, Δr must be the minimum of

$$\Delta r_1 \leq \frac{\lambda}{\delta\left(\frac{c'(z)c_0}{c(z)^2}\right)_{max}}$$

and

$$\Delta r_2 \leq \frac{z_{max}}{\tan\theta_{max}} \; .$$

In the SAIC code there is presently a constraint on the range step

$$\Delta r_3 \leq \frac{\lambda}{1-\cos\theta_{max}}$$

which is a consequence of the heuristic argument that the range step should not exceed the interference length of the fields propagating horizontally and at the maximum angle. Furthermore, since Δr_3 could become large without bound for low frequencies and angles, Δr_3 is restricted to 0.5 nm. Although this latter restriction is arbitrary, it serves to control (in a way not well-quantified) the errors induced by factoring the exponential operator.

It is interesting to note that the range-step constraint Δr_3 is actually related to Δr_1 and, therefore, has a basis more fundamental than the heuristic argument of interference lengths. To see this, consider the case where a constant-gradient sound speed in the ocean bottom is used to define the maximum angle of propagation:

$$c(z) = c_o + c'z$$

and

$$c_{max} = c_o + c'\delta$$

or

$$c'\delta = c_{max} - c_o$$

The denominator of Δr_1 is then approximated by

$$\delta\left(\frac{c'(z)c_o}{c(z)^2}\right) \approx \frac{(c_{max} - c_o)}{c_{max}}$$

$$= 1 - \frac{c_o}{c_{max}}$$

$$= 1 - \cos\theta_{max}$$

where the reference sound speed has been taken as the sound speed at the ocean bottom. With these approximate expressions, we have

$$\Delta r_1 \approx \frac{\lambda}{1-\cos\theta_{max}} .$$

It appears, then, that the constraint Δr_1 is not significantly different from the present constraint Δr_3.

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